Tile Reduction: an OpenMP Extension for Tile Aware Parallelization

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Abstract

Tiling is widely used by compilers and programmers to optimize scientific and engineering code for better performance. Many parallel programming languages support tile/tiling directly through first-class language constructs or library routines. However, the current OpenMP programming language is tile oblivious, although it is the de facto standard for writing parallel programs on shared memory systems. In this paper, we introduce tile aware parallelization into OpenMP. We propose tile reduction, an OpenMP tile aware parallelization technique that allows reduction to be performed on multi-dimensional arrays. The paper has three contributions: (a) it is the first paper that proposes and discusses tile aware parallelization in OpenMP. We argue that, it is not only necessary but also possible to have tile aware parallelization in OpenMP; (b) the paper introduces the methods used to implement tile reduction, including the required OpenMP API extension and the associated code generation techniques; (c) we have applied tile reduction on a set of benchmarks. The experimental results show that tile reduction can make parallelization more natural and flexible. It not only can expose more parallelism in a program, but also can improve its data locality.

1 Introduction

Tiling [1][2] has been used as an effective compiler optimizing technique to generate high performance scientific codes. Tiling not only can improve data locality for both the sequential and parallel programs [3], but also can help the compiler to maximize parallelism and minimize synchronization [4] for programs running on parallel machines. Thus, sometimes, it is used by the programmers to hand-tune their scientific programs to get better performance.

Tiling is essentially a program design paradigm. It is a natural representation for many important data objects that are heavily used in scientific and engineering algorithms. Scientific code that is written with the concept of tile/tiling in mind usually looks concise and clear, and thus is much easier to understand and less error prone.

Due to these advantages, it is desirable to provide certain high level language constructs in the programming languages to support tile/tiling in program design directly. To meet this requirement, researchers have proposed various designs in many parallel programming languages or sublanguages. The examples include HPF[5], UPC[6], X10[7], ZPL[8], CAF[9], Titanium[10], and HTA[11], which are among the most popular parallel languages. However, it is interesting to find out that, in the current OpenMP APIs, no directive or clause can be used to annotate data tiles and carry such information to the OpenMP compiler. In other words, the current OpenMP programming language is tile oblivious, although it is the de facto standard for writing parallel programs on shared memory systems.

In this paper, we propose tile aware parallelization for the OpenMP programming language. Its purpose is to enhance the OpenMP API with the concept of tile/tiling so that more data parallelism can be exposed to the OpenMP compiler. Besides granting greater flexibility to the OpenMP compiler to perform more data parallelization, it brings better data locality into the code. This is achieved by extending the current OpenMP directives, clauses, and runtime routines, or introducing new language constructs into OpenMP. Our first effort in this direction is termed tile reduction, an OpenMP tile aware parallelization technique that allows parallel reduction to be performed on multi-dimensional arrays.
Reduction is a form of recursive calculation that use mathematically associative and commutative operators to "aggregate" a set of data. Reduction can be performed in parallel to improve performance. For this reason, many programming languages and sub-languages support parallel reduction. Some examples are UPC [12], MPI [13], ZPL [14], and OpenMP [15]. According to the current OpenMP API specification, reduction can only be performed on "named scalar" variables. It cannot be applied on multi-dimensional arrays. We call this kind of reduction scalar reduction. In this paper, we introduce a new technique called tile reduction, which evolves the current reduction parallelization from scalar variables to multi-dimensional arrays. We have extended the traditional reduction clause to allow the programmers to annotate their code where tile reduction can be applied. We have also developed the required code generation technique to interpret the new reduction clause and generate the required parallel code accordingly. The major contributions of this paper are:

1. As far as the authors are aware, this is the first paper that proposes and discusses tile aware parallelization in OpenMP. We argue that, it is not only necessary but also possible to have tile aware parallelization techniques in OpenMP.

2. The paper introduces tile reduction, an OpenMP tile aware parallelization technique that applies reduction on multi-dimensional arrays. We discuss the methods used to implement tile reduction, including the required OpenMP API extension and the associated code generation technique.

3. We evaluate the tile reduction technique with a set of benchmarks. The experimental results show that using tile reduction can make the code parallelization more natural and flexible. It not only can expose more parallelism in the program but also can improve its data locality.

The rest of the paper is organized as follows. In Section 2, we use a motivating example to show why tile reduction is necessary. Section 3 will discuss how to implement tile reduction in the OpenMP compiler. We present our experimental data in Section 4 and make our conclusions in Section 5.

2 Motivation

In this section, we use the "histogram reduction" [16] code as an example to demonstrate the limits of the current OpenMP reduction clause. We will also use the same example to show the advantages of extending scalar reduction to tile reduction.

![Figure 1: The Histogram Reduction Example](image-url)
Figure 1(a) shows the code of the histogram reduction program. The code works on \( A[][][] \), a 3-dimensional array with each element containing an 8-byte \texttt{long long}. It aggregates all elements along the \( k \) dimension and stores the results in the \( 2x2 \) tile \( A[0][][] \). The diagram in Figure 1(b) shows these operations. We assume that the cache line size is 32 bytes and that the array is stored in a row-major order in the memory. Therefore, elements with the same \( k \) coordinate can be fed into the same cache line, as shown in Figure 1(c). There are three nested loops in the code. Each loop traverses one of the \( i, j, k \) dimension of the array. Data dependence only exit in loop \( k \) because of the recursive calculation.

(a) Parallelize loop "i"  
(b) Parallelize loop "i" and "j" using the collapse clause

Figure 2: Parallelize the Histogram Reduction Program Without Changing the Code

Given the code in Figure 1(a), there are many different ways to parallelize it. However, due to the data dependence in loop \( k \), we cannot parallelize this loop. Therefore, without changing the code, we can only parallelize loop \( i \) and \( j \), as shown in Figure 2(a) and 2(b). It is obvious that there are trivial workload and little parallelism in loop \( i \) and loop \( j \). Thus, it is not worthwhile to parallelize these two loops, even while using the \texttt{collapse} clause (supported in OpenMP 3.0 [15]).

(a) Parallelize the outer loop  
(b) Parallelize the outer two loops

(c) Nested parallelization to harvest more parallelism  
(d) Data access pattern

Figure 3: More Parallelization for Histogram Reduction Code

To get a larger workload and more parallelism, we can interchange the loops manually before parallelizing the code, as shown in Figure 3. In Figure 3(a) and 3(b), the workload that can be assigned to the threads is large enough. However, the available parallelism is still very small (only supports two
or four concurrent threads). Figure 3(c) shows a better solution. In Figure 3(c), a nested parallel directive is used to parallelize the recursive addition using the reduction clause (with trivial code change). Although the code in Figure 3(c) can leverage all levels of parallelism in the program, its strided data access pattern would cause a great number of unnecessary cache misses, as shown in Figure 3(d). Code in Figure 3(a) and 3(b) have the same data locality problem. Apparently, the current OpenMP parallelization techniques cannot harvest the maximum parallelism and data locality in the code at the same time. They suffer from either insufficient parallelism or poor data locality.

![Figure 4: The Ideal Parallelization Schema for the Histogram Reduction Code](image)

The ideal parallelization is shown in Figure 4. Logically, the recursive addition can be viewed as being performed on an array of 2x2 data tiles. In theory, these tiles can be added together in parallel by multiple threads, as shown in Figure 4(a). In this way, the code can achieve both the maximum parallelism and the best data locality (see Figure 4(b)). Besides, from the programmers’ angle, this is the most natural way to perform parallelization on this piece of code. However, the current OpenMP specification does not provide any mechanism to support such kind of parallelization. This motivates us to extend the traditional scalar reduction to tile reduction.

## 3 Tile Reduction

In this section, we will discuss the techniques used to implement tile reduction. They include the extended OpenMP programming interface and the required code generation design. The related runtime support will be mentioned when needed.

### 3.1 Programming Interface Extension

In order to support tile reduction, we need to extend the current OpenMP programming interface. The extension was made based on three criteria. First, it must be able to cover most of the common cases of tile reduction code. Second, it must be simple and easy to use and provide the programmers with the maximal flexibility. Third, the extension should not complicate the code generation of the OpenMP compiler and the OpenMP runtime. Figure 5(a) shows the OpenMP API (C/C++) extension we proposed for the reduction clause. Figure 5(b) gives a simple example that uses the extended reduction clause to parallelize the tile reduction code.
Compared with the current OpenMP API specification, the difference is in the list construct. In addition to the "named scalar" variables, we allow the programmers to put a "multi-dimensional array" in the list construct. This "multi-dimensional array" is not a real array data structure in the language sense. It is a language construct that conveys important information to the OpenMP compiler. It tells the compiler the shape, the size, and the element type of the tile and how its elements are traversed by the loops.

To make the paper easy to follow, we call the tile under reduction as the reduction tile; the "multi-dimensional array" in the list construct as the tile descriptor; and the loops involved in performing "one" recursive calculation as the reduction kernel loops. For the example in Figure 5(b), the reduction tile is B[2][2], the tile descriptor is B[j,0,2][i,0,2], and the reduction kernel loops are the j and i loops (not including the k loop, i.e., the parallelized loop). In our design, the shape of the reduction tile must be a rectangle or a high-dimensional rectangle. Triangle or other shapes are not yet supported. The exact shape and size of the reduction tile are determined by the tile descriptor.

The format of the tile descriptor is shown in Figure 5(a). It has two parts: the tile name (i.e., T) and the dimension descriptor (i.e., [j_k,L_k,U_k][j_2,L_2,U_2][j_1,L_1,U_1]). Tile name must be the same as the multi-dimensional array variable on which the recursive calculations are performed. For the example in Figure 5(b), this corresponds to the name of the lhs variable in line 4, which is B. It tells the OpenMP compiler the data type of the tile element, which must be a built-in scalar type. The dimension descriptor, on the other hand, is an array of 3-tuples. Each 3-tuple corresponds to one dimension of the tile and stores important information of that dimension. These 3-tuples are listed in the dimension descriptor in descendant order (higher dimension first). Each 3-tuple has three elements: loop index variable, upper bound expression, and lower bound expression. The loop index variable identifies a loop in the reduction kernel loops. Since stride accesses are not allowed, the loop stride is always 1, so it is
The size of the \( k \)-dimensional tile is calculated from equation (1).

\[
(U_k - L_k) \times ... (U_2 - L_2) \times (U_1 - L_1)
\]  \hspace{2cm} (1)

The information stored in the tile descriptor is very important for the OpenMP compiler to generate correct parallel code.

The operator, as usual, must be a mathematically associative and commutative operator that performs the recursive calculation. In our current example, it is a “+”.

```
0 #pragma omp parallel for reduction(+: A[0,0,2][i,0,2])
1 for (k=1; k<1000000; k++)
2 for (j=0; j<2; j++)
3 for (i=0; i<2; i++)
4 A[0][j][i] += A[k][j][i]
```

Figure 6: Tile reduction: tile is part of a bigger multi-dimensional array

The reduction tile is not required to be a standalone multi-dimensional array. Instead, it can be part of another larger multi-dimensional array. For example, in the code in Figure 6, the reduction tile is \( A[0][j][i] \) \( (j = \{0,1\}, i = \{0,1\}) \). It is a \( 2 \times 2 \) slice cut out from the 3-dimensional array \( A[][][] \).

Besides, as we have mentioned before, the lower and upper bounds in the dimension descriptor are expressions. They are not required to be constants. Generally, the lower and upper bounds can be a function of other variables, as long as the result of the function can be decided at runtime. Figure 7 shows such an example. The code in Figure 7 is a blocked matrix multiplication program. It is easy to see that there is an opportunity to apply tile reduction on the loop in line 3, i.e., the \( kk \) loop. The diagram on the right hand side gives an intuitive illustration. In this example, the reduction tiles are blocks cut out from a big \( 2 \times 2 \) matrix \( C[][] \). Therefore, the lower and upper bounds of the reduction tiles are not fixed values. In addition, the matrix \( C[][] \) might not be able to be evenly blocked. So, the tiles located at the margin of the matrix are usually smaller than the tiles located inside of the matrix. Thus, the sizes of the reduction tiles are not necessarily the same. All these information is reflected in the lower and upper bound expressions (or functions) in the dimension descriptor. Moreover, there is a restriction for the lower bound and upper bound expressions. They should not be functions of any index variable in the reduction kernel loops, i.e., they are orthogonal. This is to make sure that the shape of the reduction tile is a rectangle, or high-dimensional rectangle.

An interesting observation of this example code is that the number of the reduction kernel loops (which is 3, from line 4 to line 6) is not the same as the dimension of the reduction tile (which is 2). Generally, we do not require the number of the reduction kernel loops to be the same as the dimension of the reduction tile. We only require that the operations performed by the code in the reduction kernel loops can be viewed as one associative and commutative macro operation performed on the entire reduction tile.
3.2 Code Generation

Since tile reduction is derived from scalar reduction, its code generation shares the same framework as scalar reduction. Thus, we illustrate the code generation for tile reduction under the same framework as scalar reduction and use the code generation for scalar reduction as a reference. Generally, the code generation needs to deal with the following problems:

1. Distribute the iterations of the parallelized loop among the threads;
2. Allocate memory for the private copy of the tile used in the local recursive calculation;
3. Perform the local recursive calculation which is specified by the reduction kernel loops;
4. Update the global copy of the reduction tile;

Figure 8 shows the code generated for the tile reduction example in Figure 7. To make the paper easy to follow, we present the pseudo C code in the figure.

As we have mentioned at the beginning of Section 3.1, we try to avoid complicating the code generation when we were developing the extension for the reduction clause. A good example is the code generation for distributing the iterations of the parallelized loop among the dynamic threads. Actually, this part of the code generation for tile reduction is the same as that for scalar reduction.

In the tile reduction program, the reduction kernel loops can be viewed as a single statement that performs the recursive calculation, which is the same as its counterpart in the scalar reduction program. So, from the angle of iteration distribution, the scalar reduction code and the tile reduction code are logically the same. Therefore, the method used to generate iteration distribution code for scalar reduction can also be used to generate iteration distribution code for tile reduction. It doesn’t matter which schedule policy (static, dynamic, guided, or runtime) is deployed.

In Figure 8, we use the static scheduling policy as an example. In the code from line 2 to line 6, the iterations of the \( kk \) loop (line 3 in Figure 7) are evenly distributed among the threads. The iterations of the loop are divided into chunks and each chunk is assigned to one dynamic thread. The iteration chunk assigned to the thread is delimited by the lower bound variable \( \text{lb} \) and the upper bound variable \( \text{ub} \), which are determined by the thread number of the owner thread. This piece of
# Matrix Multiplication with Tile Reduction

This figure illustrates the pseudocode generated for the matrix multiplication example to perform tile reduction. The code only deals with the parallelized loop and the user specified OpenMP parameters. It does not even need to look into the code of the reduction kernel loops. This is the same for other schedule policies.

In line 9, the OpenMP runtime routine allocates memory for the private tile (private_tile), which is a 2-dimensional array. This private tile is used by the thread as a temporary storage to perform the local sequential tile reduction. Its size is calculated from the parameters specified in the dimension descriptor (see equation 1). Its element data type is inferred from the tile name. All this information is obtained from the extended reduction clause.

The local sequential tile reduction is performed in the code from line 12 to line 16. This piece of code is almost the same copy as the original sequential program (line 3 to line 7 in Figure 7) except two places. In line 12, the lower and upper bounds of the loop are changed to "lb" and "ub". This is to restrict the range of the iteration space in the chunk assigned to the current thread. Besides, in line 16, we replace the original reduction tile with the private tile and update its indices. This index calibration is required because the global reduction tile is cut out from a bigger multi-dimensional array, while the private tile is a standalone array. This piece of code performs local tile reduction sequentially, as in the original un-parallelized code.

After finishing the local tile reduction, the thread must update the global reduction tile. The code is shown in line 19 to line 23. The runtime routines in lines 19 & 23 ensure atomic access to the global...
reduction tile. The loops in line 20 and line 21 are extracted from the reduction kernel loops. Only the loops listed in the tile descriptor are selected. So, the loop $k$ in the reduction kernel loops is not included. The $l_{hs}$ variable of the statement in line 22 is the same variable as in the original code (line 7 in Figure 7). However, the $r_{hs}$ variable has been replaced with the private tile, in which the indices have been updated.

From the code in Figure 8, it is easy to see that the code generation for the tile reduction is as easy as that for the traditional scalar reduction. Meanwhile, no extra runtime supports is required. These advantages make the implementation of tile reduction in the OpenMP compiler very easy. In the next section, we will present the experimental results of applying the tile reduction on several typical benchmarks.

4 Experiments

We have applied tile reduction on three benchmarks: the 2D histogram reduction, matrix-matrix multiplication and matrix-vector multiplication. The required code generation was implemented through source-to-source transformation and was prototyped in the Omni-1.6 OpenMP compiler [17]. The machine used in the experiments has 4 Intel Dual-Core Xeon (Paxville) chips, which are clocked at 3.0 GHz. Each core has HyperThreading (HT) enabled. Therefore, the machine can be viewed as a 16-processor shared memory parallel computer. Each chip has 4MB L2 cache (2MB each core) and each core has 16KB L1 cache.

Figure 9 shows the experimental data of the three benchmarks. The curve graphs on the left column display the speedup of the benchmark programs parallelized either through the tile reduction clause (w/ tile reduction) or through the standard OpenMP APIs (w/o tile reduction). The bar charts on the right column demonstrate the difference of the absolute execution time between the corresponding programs (w/ and w/o tile reduction) of the same set of benchmarks.

Figure 9(b) shows great performance enhancement if we parallelize the 2D histogram reduction benchmark with the tile reduction clause. Generally, compared with the program parallelized with standard OpenMP pragma, the absolute execution time of the tile reduction version decreased about 90% and its speedup on 8 threads increased from 1.5 to 4.5. The performance gain comes from the improved data locality, which owes to the tile reduction optimization. Without using tile reduction, the 2D histogram reduction program exhibit very poor scalability (shown in Figure 3). The tile reduction parallelization successfully rectifies the data access pattern and thus significantly improves its scalability. However, no matter what kind of optimizations are used, this benchmark stops scaling beyond 8 threads. This is because of the huge number of memory references in the code, which results in that its performance is finally restricted by the bandwidth of the shared memory bus.

The same phenomena are also observed in the matrix-matrix multiplication benchmark (see Figure 9(c) and 9(d)). Tile reduction can also decrease its execution time and improve its scalability. However, the magnitude of the performance enhancement caused by tile reduction is not as big as that of the 2D histogram reduction benchmark. This is also the same for the scalability enhancement. The reason is that the data locality of the tiled matrix-matrix multiplication program is better than the 2D histogram
Figure 9: Comparison of the speedup and execution time between the program parallelized with tile reduction and the program parallelized with the standard OpenMP pragma.

reduction benchmark. Therefore, the performance gain from tile reduction in the matrix multiplication program is less than that in the 2D histogram reduction program. On average, the execution time decreased 34% after applying tile reduction and its speedup increased from 2.15 to 3.18 on 8 threads and from 2.26 to 3.32 on 16 threads.

For the matrix-vector multiplication case, the performance enhancement brought about by tile re-
duction is smaller than that of the previous two benchmarks. The reason is the same as the previous one. Moreover, compared with the other two benchmarks, there are less data memory references in this benchmark. So, the program’s performance degrades a little bit when it runs with 8 or 16 threads. This is because of the synchronization overhead caused by the code in line 19 and 23 in Figure 8. In average, its execution time decreased 0.28%.

5 Summary and Conclusions

In this paper, we introduced the concept of tile aware parallelization for OpenMP. Meanwhile, we developed the first tile aware parallelization technique - tile reduction, and illustrated the details of code generation for the tile reduction clause. We also designed a series of experiments to evaluate the tile reduction technique. From the experimental results and our experience of parallelizing the benchmarks, we have the following conclusions:

1. As a building block of the tile aware parallelization theory, tile reduction brings more opportunities to parallelize dense matrix applications.

2. For some benchmarks, tile aware parallelization is a more natural and intuitive way to reason about the best parallelization decision.

3. Tile reduction not only can improve data locality for some programs, but also can expose more parallelism.

6 Related Work

Parallel reduction operations are supported in many parallel programming languages. They include C**[18], SAC [19], ZPL [16], UPC [12], and MPI [13]. Most of them support user-defined reduction operations, either through language constructs or through library routines. User-defined reduction operation provides a flexible way to implement tile reduction. However, programmers need to change both data structures and algorithms, which, sometimes, is not a trivial job.

Another piece of work that we need to mention is [20]. In [20], the authors propose to extend the OpenMP reduction clause to parallelize C++ generic algorithms. They propose to support user-defined types, overloaded operators, and function objects in the same way as the built-ins supported in the current OpenMP reduction clause. Their work is very close to that presented in this paper. However, we study the reduction problem from a different angle. We propose tile reduction as one of the tile aware parallelizing technique for OpenMP, while [20] proposes user-defined reduction operation to complete their OpenMP extensions for parallelizing generic libraries. In our tile aware parallelization technique, we are concerned with the data partition, locality and a more flexible and efficient way to parallelize dense matrix programs written in canonical C syntax, while the purpose of [20] is to allow people to parallelize programs written in modern C++ idioms such as iterators and function objects, which are not canonical C syntax. Second, due to the non-trivial dynamic overhead of the generic
techniques, generic libraries are not widely used in programming high performance scientific and engineering algorithms. Finally, there are no experimental data in [20].

7 Future Work

Tile reduction is one of the building block of the tile aware paralleization technique developed for OpenMP. One of our future work is to develop more parallelizing techniques (like tile reduction) such that OpenMP compiler can "recognize" data tiles and allow its runtime library to manipulate them. Our goal is to add tile aware parallelizing directives or clauses into the OpenMP programming interface. The purpose is to evolve OpenMP into an appropriate programming model for many-core processors with explicitly managed memory hierarchy [21], e.g. the IBM CELL [22] and the IBM Cyclops-64 [23] processor.

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