Topic 1b: Flow Analysis

Some slides come from Prof. J. N. Amaral (amaral@cs.ualberta.ca)

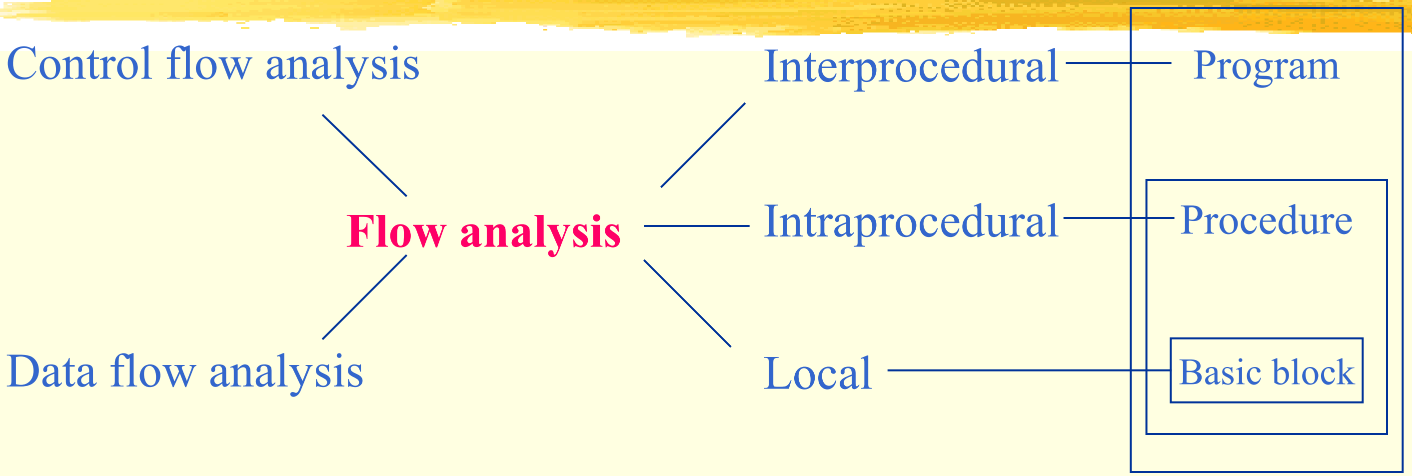
Topic 4: Flow Analysis

- Motivation
- Control flow analysis
- Dataflow analysis
- Advanced topics

Reading List

- Slides
- Dragon book: chapter 8.4, 8.5, Chapter 9
- Muchnick's book: Chapter 7
- Other readings as assigned in class or homework

Flow Analysis



- **Control Flow Analysis** — Determine control structure of a program and build a Control Flow Graph.
- **Data Flow analysis** — Determine the flow of scalar values and ceretain associated properties
- ***Solution to the Flow analysis Problem****: propagation of data flow information along a flow graph.*

Introduction to Code Optimizations

Code optimization - a program transformation that *preserves correctness* and *improves the performance* (e.g., execution time, space, power) of the input program. Code optimization may be performed at multiple levels of program representation:

1. *Source code*
2. *Intermediate code*
3. *Target machine code*

Optimized vs. optimal - the term "optimized" is used to indicate a relative performance improvement.

Motivation

S_1: A ← 2 (def of A)

S_2: B ← 10 (def of B)

⋮

S_3: C ← A + B

determine if C is a constant 12?

S_4 Do I = 1, C

A[I] = B[I] + D[I-1]

Basic Blocks

A *basic block* is a sequence of consecutive intermediate language statements in which flow of control can only *enter at the beginning and leave at the end*.

Only the last statement of a basic block can be a branch statement and only the first statement of a basic block can be a target of a branch. However, procedure calls may need be treated with care within a basic block (Procedure call starts a new basic block)

(AhoSethiUllman, pp. 529)

Basic Block Partitioning Algorithm

1. Identify *leader* statements (i.e. the first statements of basic blocks) by using the following rules:

   (i) The *first statement* in the program is a leader

   (ii) Any statement that is the *target of a branch statement* is a leader (for most IL's. these are label statements)

   (iii) Any statement that *immediately follows a branch or return statement* is a leader

2. The basic block corresponding to a leader consists of the leader, and all statements up to but not including the next leader or up to the end of the program.

Example

The following code computes the inner product of two vectors.

```
begin
   prod := 0;
   i := 1;
   do begin
         prod := prod + a[i] * b[i]
         i = i+ 1;
   end
   while i <= 20
end
```

Source code.

```
(1)   prod := 0
(2)   i := 1
(3)   t1 := 4 * i
(4)   t2 := a[t1]
(5)   t3 := 4 * i
(6)   t4 := b[t3]
(7)   t5 := t2 * t4
(8)   t6 := prod + t5
(9)   prod := t6
(10)  t7 := i + 1
(11)  i := t7
(12)  if i <= 20 goto (3)
(13)  …
```

Three-address code.

Example

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        i = i+ 1;
   end
   while i <= 20
end
```

Source code.

Rule (i)

```
(1)   prod := 0
(2)   i := 1
(3)   t1 := 4 * i
(4)   t2 := a[t1]
(5)   t3 := 4 * i
(6)   t4 := b[t3]
(7)   t5 := t2 * t4
(8)   t6 := prod + t5
(9)   prod := t6
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end
```

Source code.

Rule (i)

Rule (ii)

```
(1)   prod := 0
(2)   i := 1
(3)   t1 := 4 * i
(4)   t2 := a[t1]
(5)   t3 := 4 * i
(6)   t4 := b[t3]
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end
```

Source code.

Rule (i)	**(1) prod := 0**
	(2) i := 1
Rule (ii)	**(3) t1 := 4 * i**
	(4) t2 := a[t1]
	(5) t3 := 4 * i
	(6) t4 := b[t3]
	(7) t5 := t2 * t4
	(8) t6 := prod + t5
	(9) prod := t6
	(10) t7 := i + 1
	(11) i := t7
	(12) if i <= 20 goto (3)
Rule (iii)	**(13) …**

Three-address code.

Example

Basic Blocks:

B1

```
(1)  prod := 0
(2)  i := 1
```

B2

```
(3)  t1 := 4 * i
(4)  t2 := a[t1]
(5)  t3 := 4 * i
(6)  t4 := b[t3]
(7)  t5 := t2 * t4
(8)  t6 := prod + t5
(9)  prod := t6
(10)  t7 := i + 1
(11)  i := t7
(12)  if i <= 20 goto (3)
```

B3

```
(13)  …
```

Transformations on Basic Blocks

- Structure-Preserving Transformations:
  - common subexpression elimination
  - dead code elimination
  - renaming of temporary variables
  - interchange of two independent adjacent statements
  - Others …

Transformations on Basic Blocks

The DAG representation of a basic block lets compiler perform the code-improving transformations on the codes represented by the block.

Transformations on Basic Blocks

Algorithm of the DAG construction for a basic block

- Create a node for each of the initial values of the variables in the basic block
- Create a node for each statement ***s***, label the node by the operator in the statement ***s***
- The children of a node ***N*** are those nodes corresponding to statements that are last definitions of the operands used in the statement associated with node ***N***.

Tiger book pp533

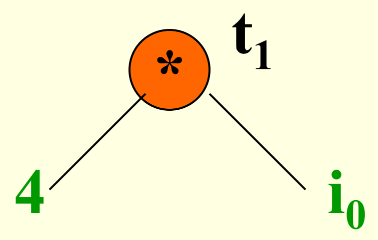
An Example of Constructing the DAG

t1: = 4*i

t2 := a[t1]

t3 := 4*i

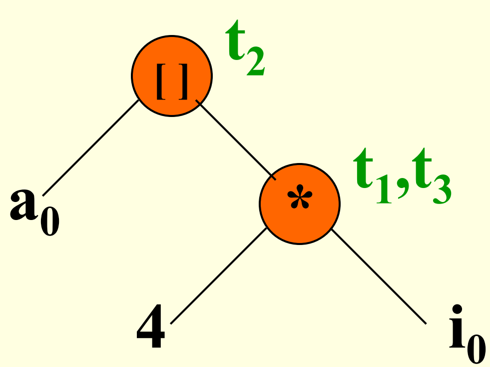
Step (1): create node 4 and i_0
Step (2): create node *
Step (3): attach identifier t_1



Step (1): create nodes labeled [], a
Step (2): find previously node(t_1)
Step (3): attach label

Here we determine that:
node (4) was created
node (i) was created
node (*) was created

just attach t3 to node *



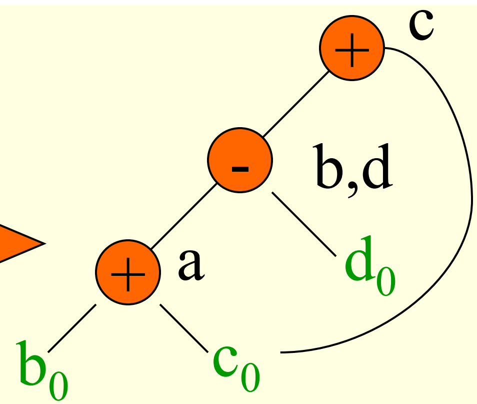
Example of Common Subexpression Elimination

(1) a:= b + c

(2) b:= a – d

(3) c:= b + c

(4) d:= a - d



a:= b + c

b:= a – d

c:= b + c

d:= b

If a node N represents a common subexpression, N has more than one attached variables in the DAG.

Detection:

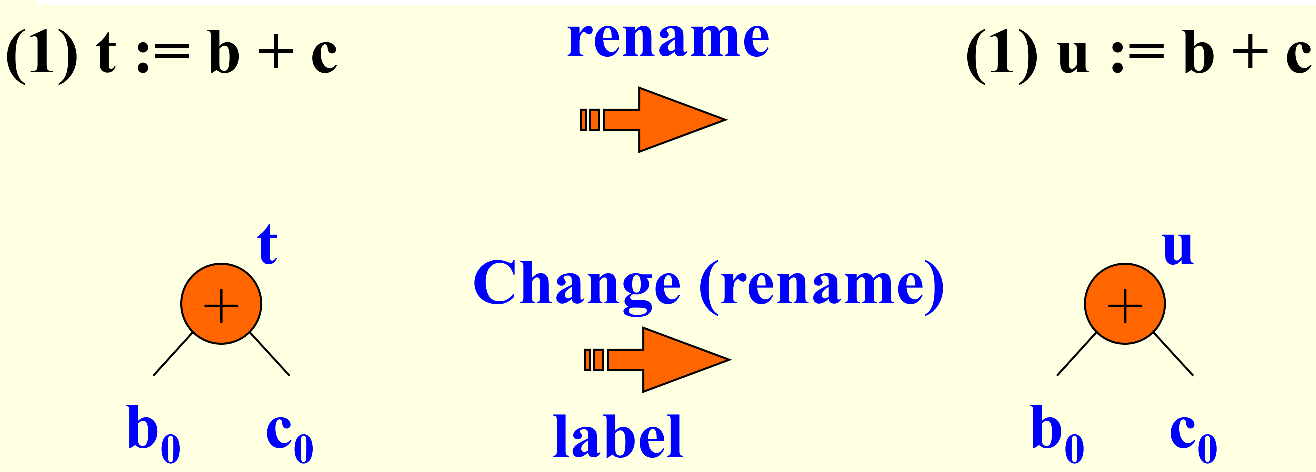
Common subexpressions can be detected by noticing, as a new node *m* is about to be added, whether there is an existing node *n* with the same children, in the same order, and with the same operator.

if so, *n* computes the same value as *m* and may be used in its place.

Example of Dead Code Elimination

if x is never referenced after the statement x = y+z, the statement can be safely eliminated.

Example of Renaming Temporary Variables

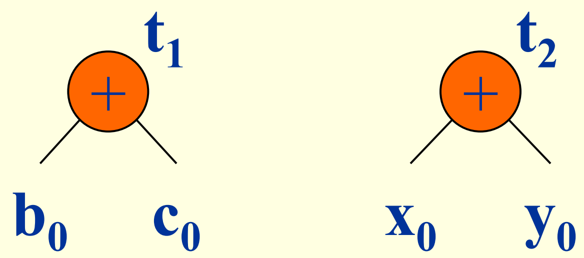


if there is an statement `t := b + c`, we can change it to `u := b + c` and change all uses of `t` to `u`.

a code in which each temporary is defined only once is called a ***single assignment form***.

Example of Interchange of Statements

t_1 := b + c
t_2 := x + y



Observation:

We can interchange the statements without affecting the value of the block if and only if neither x nor y is t_1 and neither b nor c is t_2, i.e. we have two DAG subtrees.

Example of Algebraic Transformations

Arithmetic Identities:

x + 0 = 0 + x = x

x – 0 = x

x * 1 = 1 * x = x

x / 1 = x

- Replace left-hand side with simples right hand side.

Associative/Commutative laws

x + (y + z) = (x + y) + z

x + y = y + x

Reduction in strength:

x ** 2 = x * x

2.0 * x = x + x

x / 2 = x * 0.5

- Replace an expensive operator with a cheaper one.

Constant folding

2 * 3.14 = 6.28

-Evaluate constant expression at compile time`

Control Flow Graph (CFG)

A ***control flow graph*** (CFG), or simply a flow graph, is a directed multigraph in which the nodes are basic blocks and edges represent flow of control (branches or fall-through execution).

- The basic block whose leader is the first statement is called the *initial node* or *start node*
- There is a directed edge from basic block B1 to basic B2 in the CFG if:

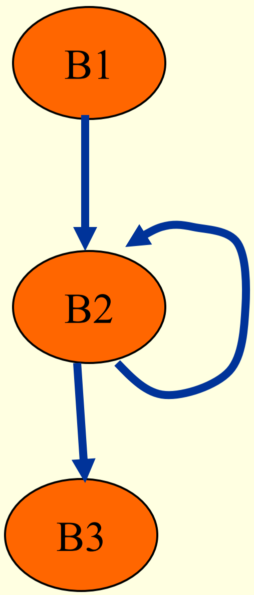
(1) There is a branch from the last statement of B1 to the first statement of B2, or

(2) Control flow can fall through from B1 to B2 because B2 immediately follows B1, and B1 does not end with an unconditional branch

And, there is an END node.

Example

Control Flow Graph:



Rule (2)

B1
```
(1)  prod := 0
(2)  i := 1
```

B2
```
(3)  t1 := 4 * i
(4)  t2 := a[t1]
(5)  t3 := 4 * i
(6)  t4 := b[t3]
(7)  t5 := t2 * t4
(8)  t6 := prod + t5
(9)  prod := t6
(10)  t7 := i + 1
(11)  i := t7
(12)  if i <= 20 goto (3)
```

Rule (2)

Rule (1)

B3
```
(13)  ...
```

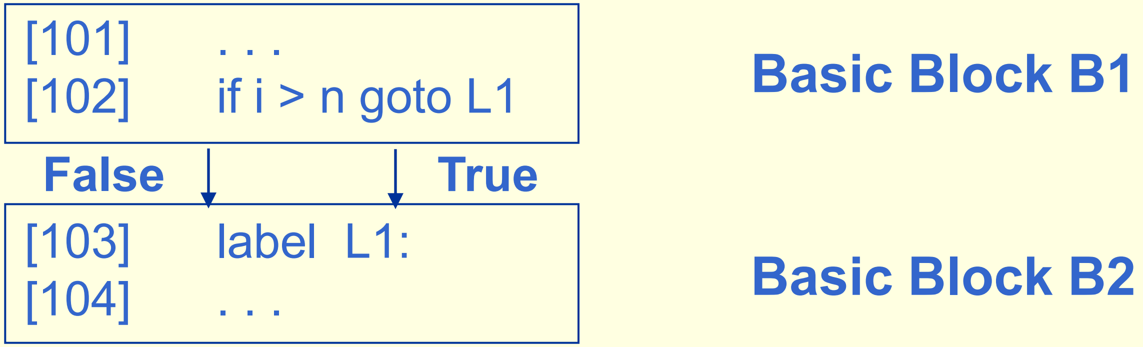
CFGs are Multigraphs

Note: there may be multiple edges from one basic block to another in a CFG.

Therefore, in general the CFG is a *multigraph.*

The edges are distinguished by their condition labels.

A trivial example is given below:



Identifying loops

Question: Given the control flow graph of a procedure, how can we identify loops?

Answer: We use the concept of dominance.

Dominators

Node (basic block) **D** in a CFG ***dominates*** node **N** if every path from the ***start node*** to **N** goes through **D**. We say that node **D** is a ***dominator*** of node N.

Define **DOM(N)** = set of node **N**'s dominators, or the ***dominator set*** for node **N**.

Note: by definition, each node dominates itself i.e., **N ∈ DOM(N).**

Domination Relation

Definition: Let ***G*** = (*N*, *E*, *s*) denote a flowgraph.
and let *n*, *n*' $\in$ *N*.

1. *n* **dominates** *n'*, written $n \leq n'$:
   each path from *s* to *n'* contains *n*.
2. *n* **properly dominates** *n'*, written $n < n'$:
   $n \leq n'$ and $n \neq n'$.
3. *n* **directly** (immediately) **dominates** *n'*, written $n <_d n'$:
   $n < n'$ and
   there is no m $\in$ *N* such that $n < m < n'$.
4. ***DOM****(n)* := $\{n' : n' \leq n\}$ is the set of dominators of *n*.

Domination Property

- The domination relation is a partial ordering
- Reflexive

  $A \leq A$

- Antisymmetric

  $A \leq B \not\longrightarrow B \leq A$

- Transitive

  $A \leq B$ and $B \leq C \longrightarrow A \leq C$

Computing Dominators

Observe: if **a** dominates **b**, then

- **a** = **b**, or
- **a** is the only immediate predecessor of **b**, or
- **b** has more than one immediate predecessor, all of which are dominated by **a**.

$$DOM(\mathbf{b}) = \{\mathbf{b}\} \cup \bigcap_{\mathbf{p} \in pred(\mathbf{b})} DOM(\mathbf{p})$$

Quiz: why here is the intersection operator instead of the union?

An Example

Domination relation:

{ (1, 1), (1, 2), (1, 3), (1, 4) …
(2, 3), (2, 4), …
(2, 10)
...
}

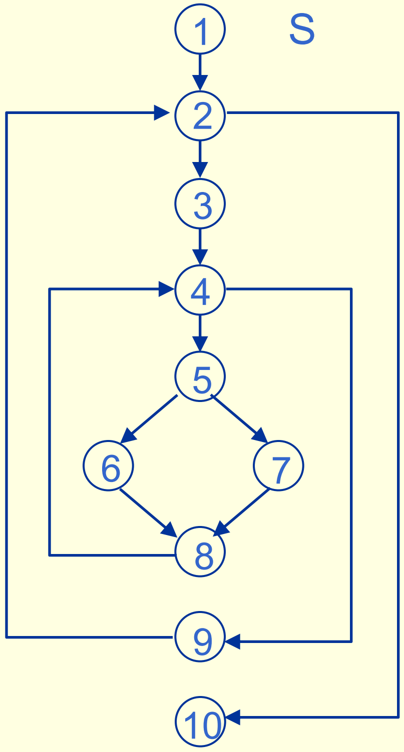
Direct domination:

$1 <_d 2,\ 2 <_d 3,\ \ldots$

DOM:

DOM(1) = {1}
DOM(2) = {1, 2}
DOM(10) = {1, 2, 10}
…

DOM(8) ?



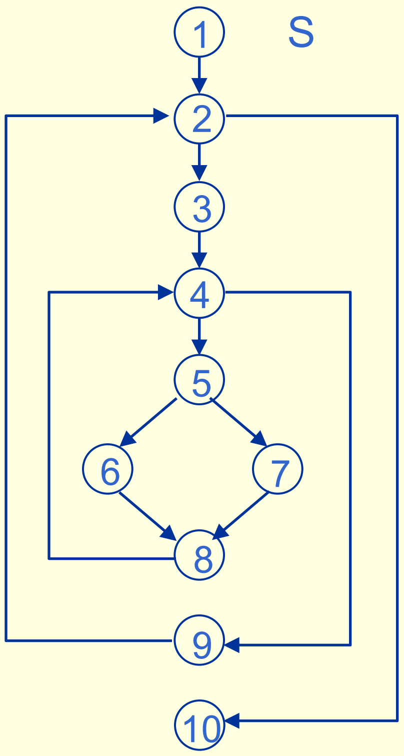
DOM(8) ={ 1,2,3,4,5,8}

Question

Assume node ***m*** is an immediate dominator of a node ***n***, is ***m*** necessarily an immediate predecessor of ***n*** in the flow graph?

Answer: ***NO!***

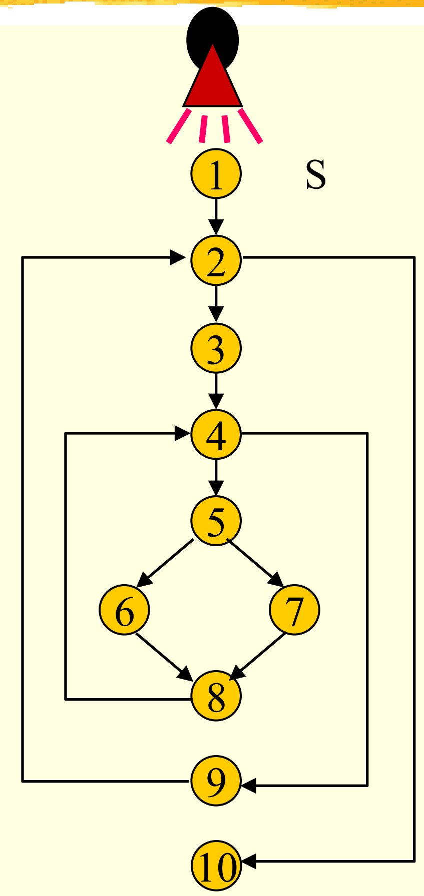
Example: *consider nodes 5 and 8.*



Dominance Intuition

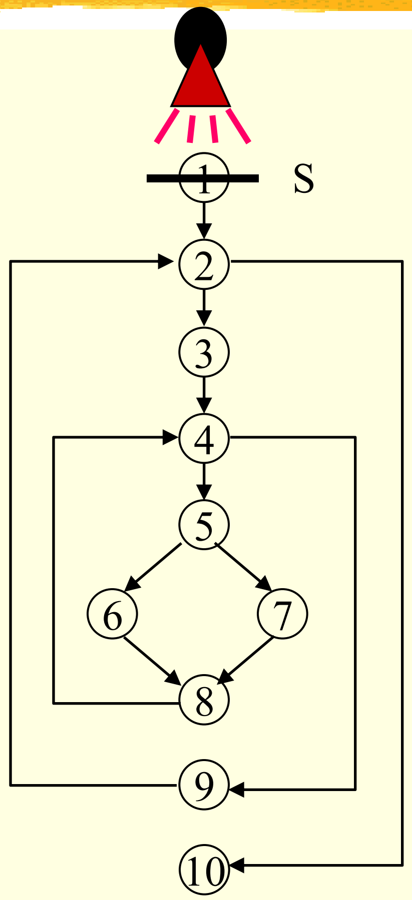
Imagine a source of light at the start node, and that the edges are optical fibers

To find which nodes are dominated by a given node **a**, place an opaque barrier at **a** and observe which nodes became dark.



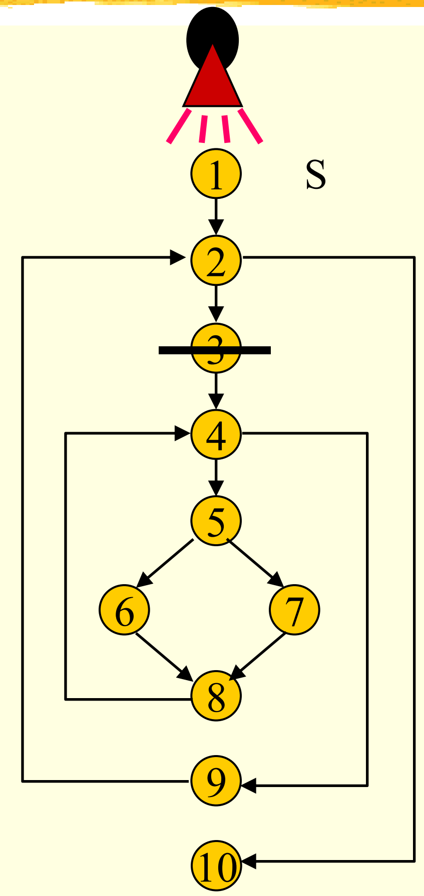
Dominance Intuition

The start node dominates all nodes in the flowgraph.



Dominance Intuition

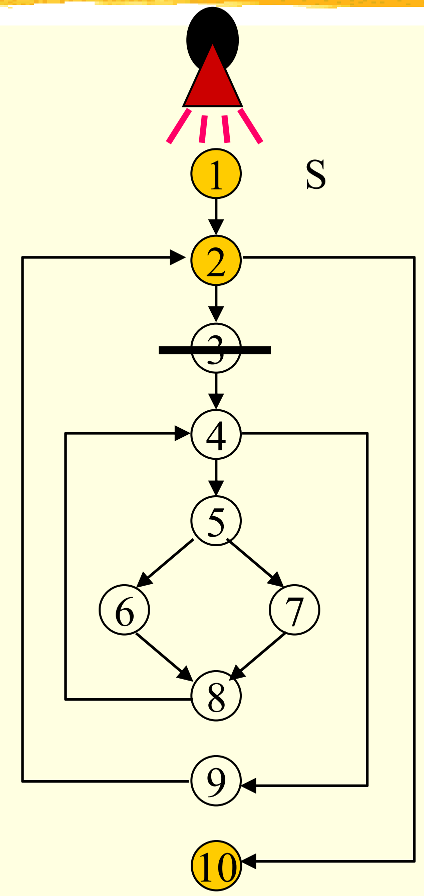
Which nodes are dominated by node 3?



Dominance Intuition

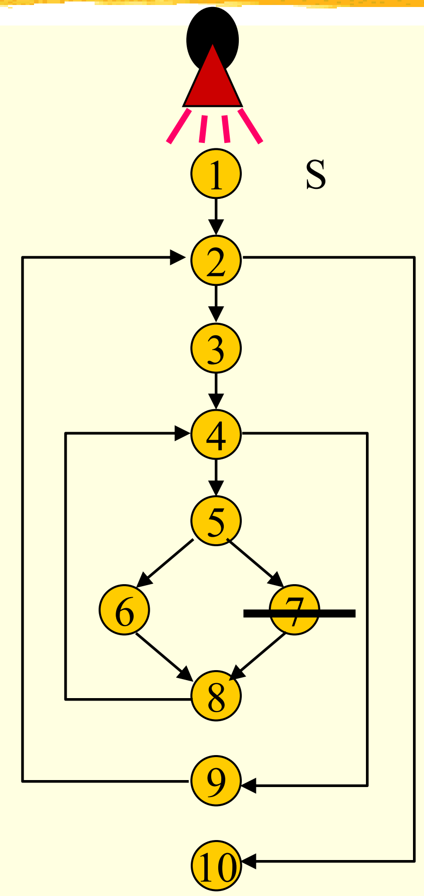
Which nodes are dominated by node 3?

Node 3 dominates nodes 3, 4, 5, 6, 7, 8, and 9.



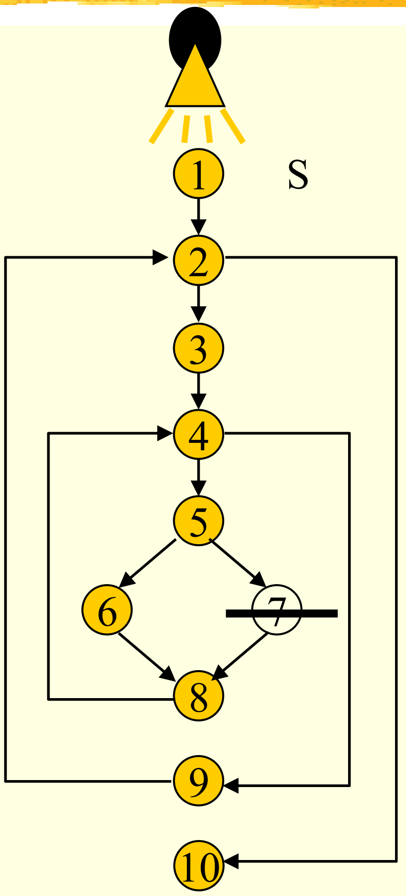
Dominance Intuition

Which nodes are dominated by node 7?



Dominance Intuition

Which nodes are dominated by node 7?



Node 7 only dominates itself.

Immediate Dominators and Dominator Tree

Node **M** is the immediate dominator of node **N** ==> Node **M** must be the last dominator of **N** on any path from the start node to **N**.

Therefore, every node other than the start node must have a *unique* immediate dominator (the start node has no immediate dominator.)

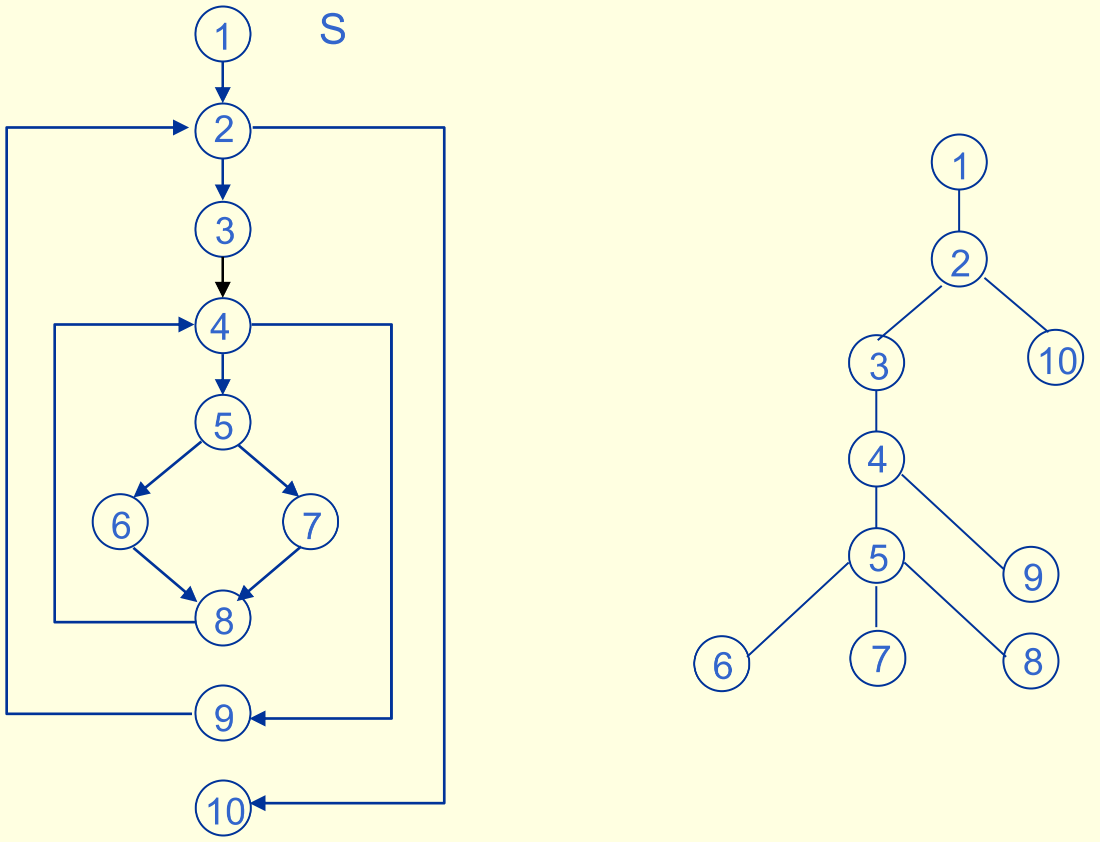
What does this mean ?

A Dominator Tree

A dominator tree is a useful way to represent the dominance relation.

In a dominator tree the start node **s** is the root, and each node **d** dominates only its descendants in the tree.

Dominator Tree (Example)



A flowgraph (*left*) and its dominator tree (*right*)

Natural Loops

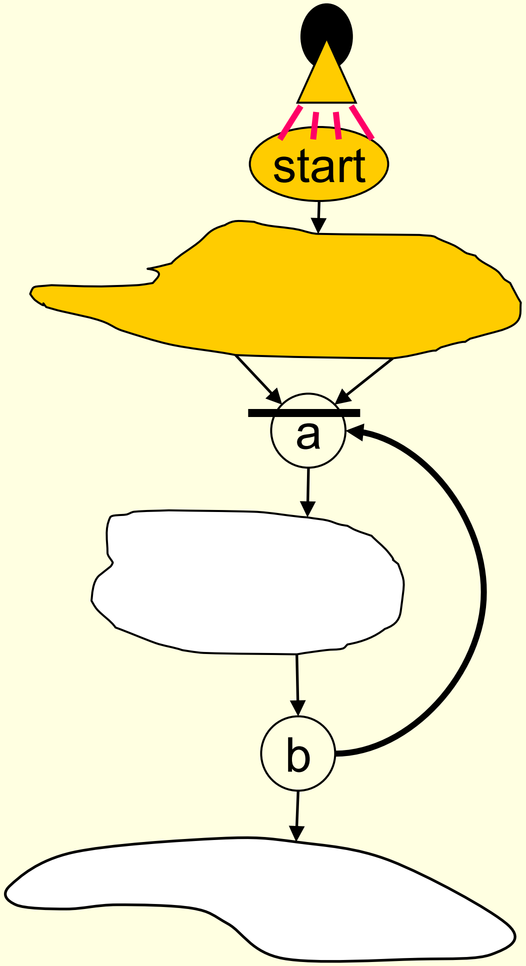
- *Back-edges* - an edge (*B, A*), such that

  A < *B* (A properly dominates B).

- Header --A single-entry node which dominates all nodes in a subgraph.

- *Natural loops*: given a back edge (B, A), a natural loop of (B, A) with *entry* node A is the graph: A plus all nodes which is dominated by A and can reach B without going through A.

Find Natural Loops



One way to find natural loops is:

1) find a back edge (b,a)

2) find the nodes that are dominated by a.

3) look for nodes that can reach b among the nodes dominated by a.

Algorithm to finding Natural Loops

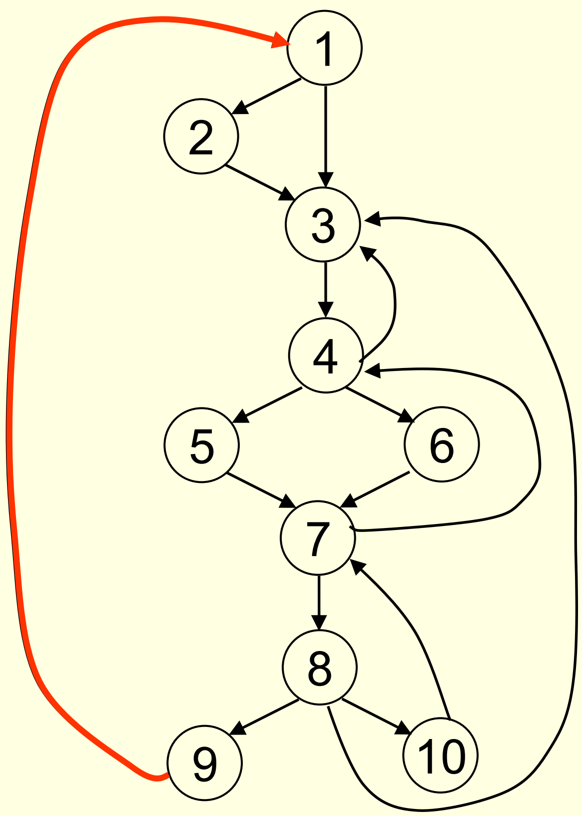
Input: A flow graph ***G*** and a back edge **n -> d**

Output: the natural loop of **n ->d**

- Initial a loop ***L*** with nodes **n** and **d**: ***L***={**n**, **d**};
- Mark **d** as "visible" so that the following search does not reach beyond **d**;
- Perform a depth-first search on the control-flow graph starting with node **n**;
- Insert all the nodes visited in this search into loop ***L***.

- Alg. 9.46 (Aho et. al., pp665)

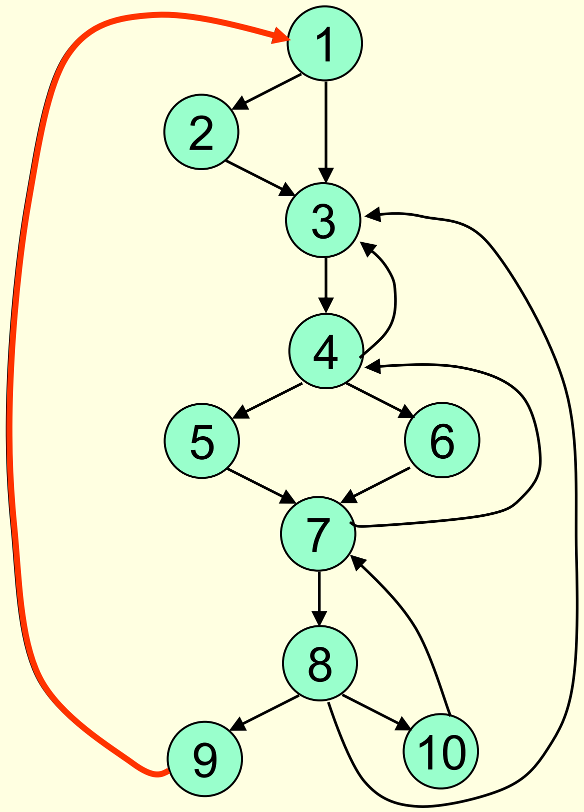
An Example



Find all back edges in this graph and the natural loop associated with each back edge

Back edge	Natural loop
(9,1)	

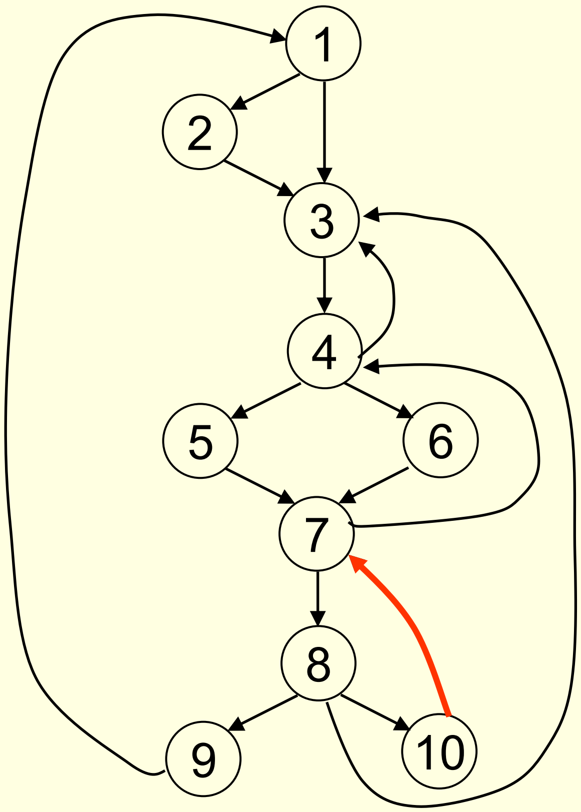
An Example



Find all back edges in this graph and the natural loop associated with each back edge

Back edge	Natural loop
(9,1)	Entire graph

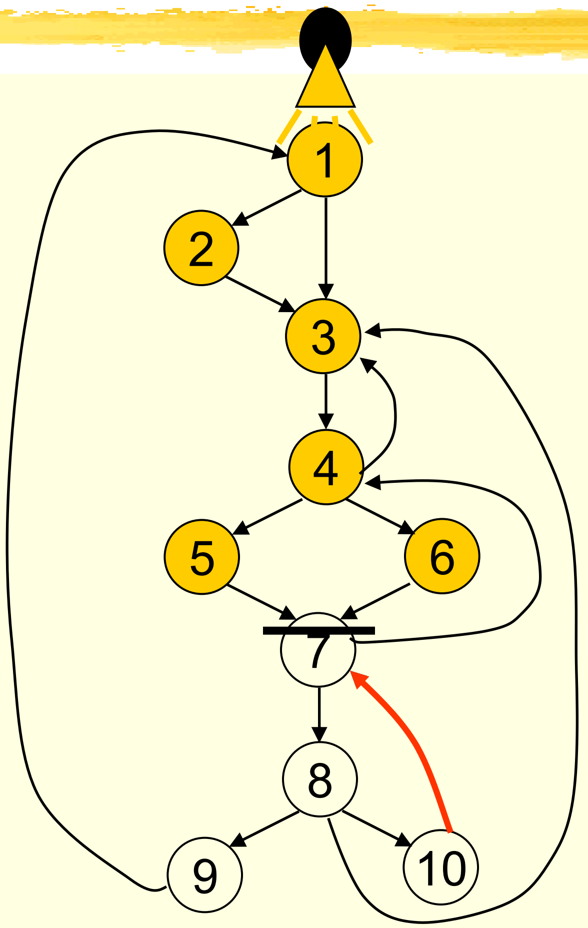
An Example



Find all back edges in this graph and the natural loop associated with each back edge

Back edge	Natural loop
(9,1)	Entire graph
(10,7)	

An Example



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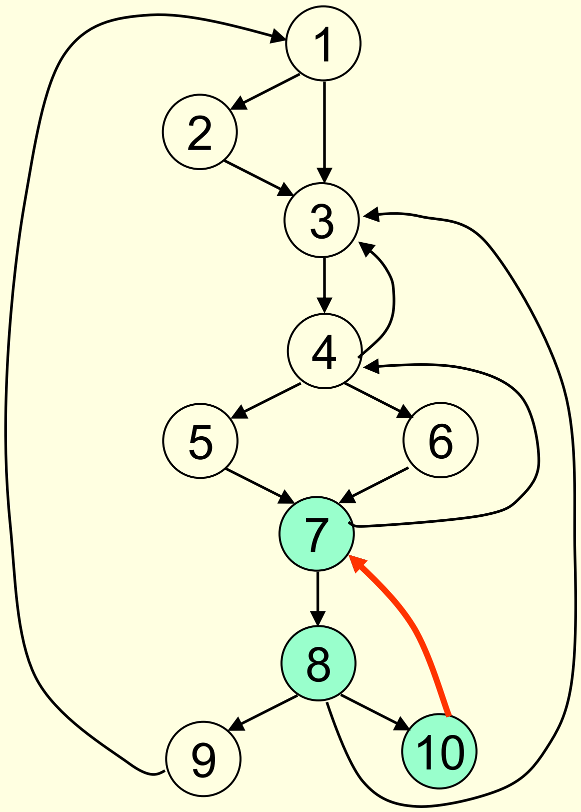
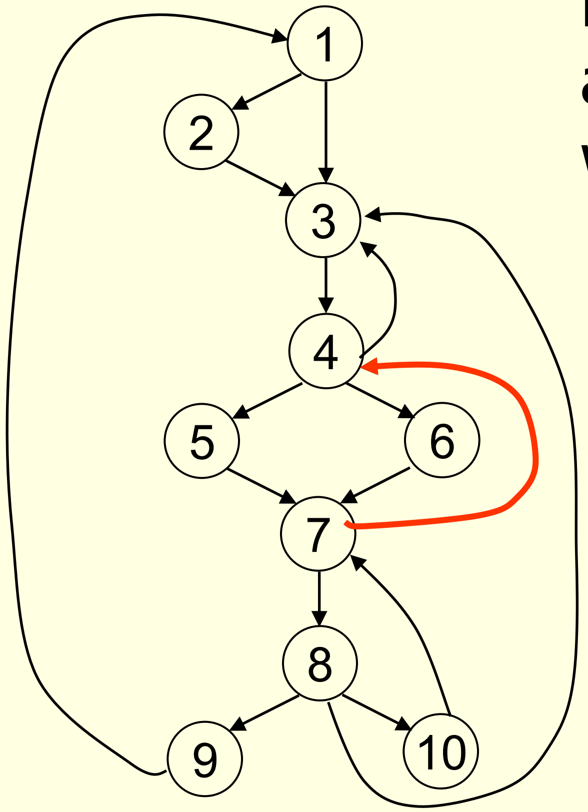


Find all back edges in this graph and the natural loop associated with each back edge

Back edge	Natural loop
(9,1)	Entire graph
(10,7)	{7,8,10}

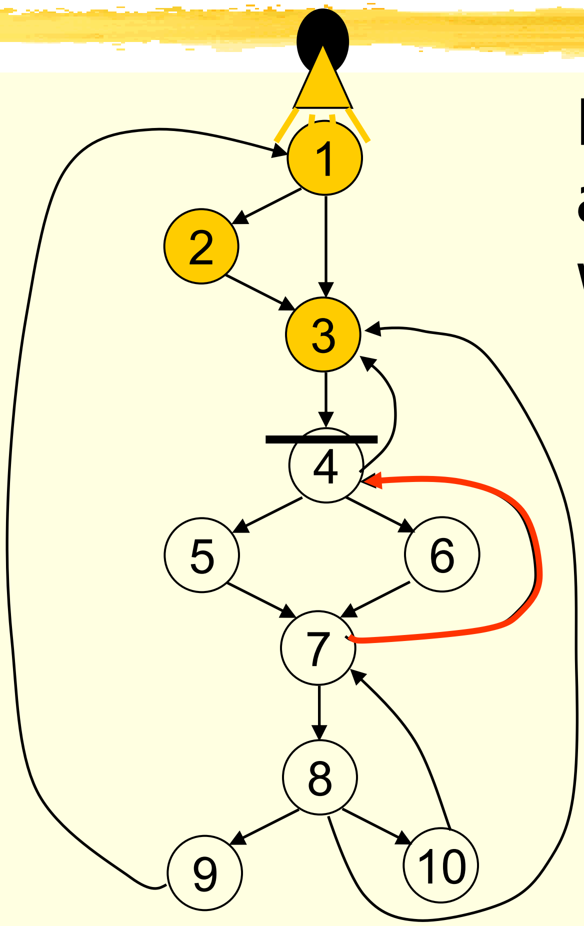
An Example



Find all back edges in this graph and the natural loop associated with each back edge

Back edge	Natural loop
(9,1)	Entire graph
(10,7)	{7,8,10}
(7,4)	

An Example



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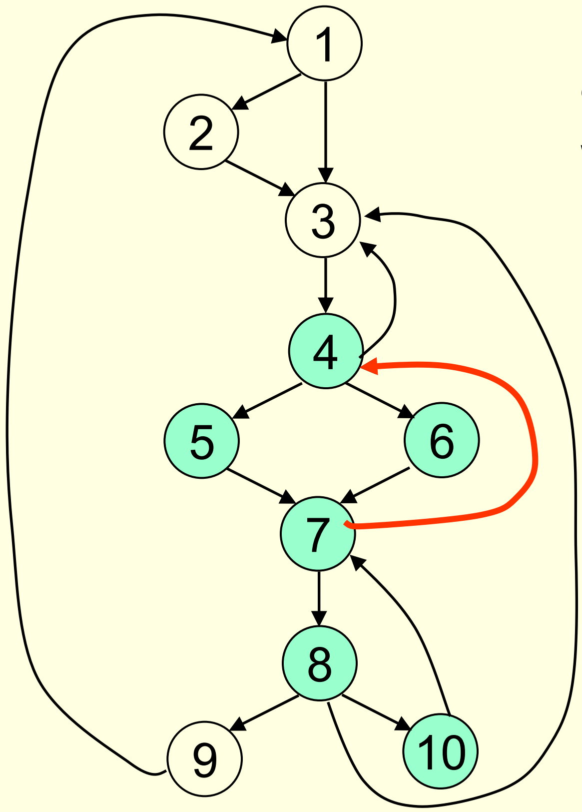

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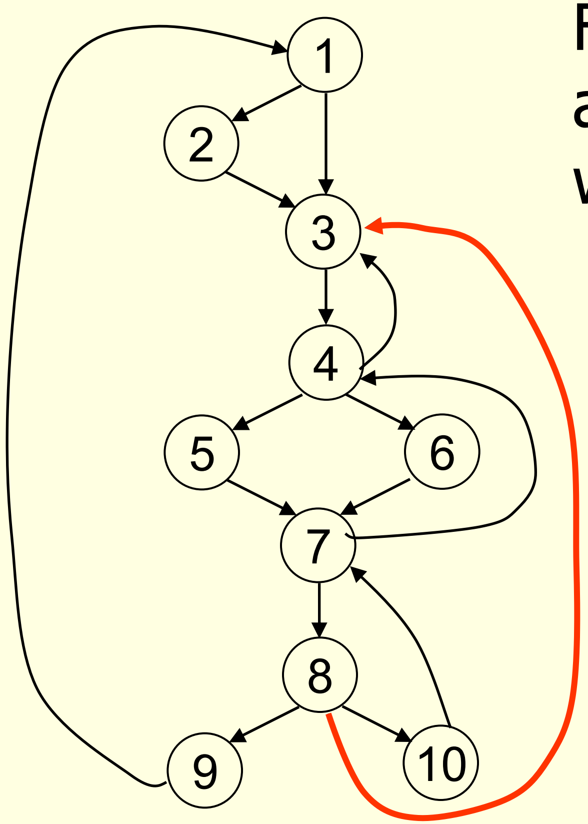

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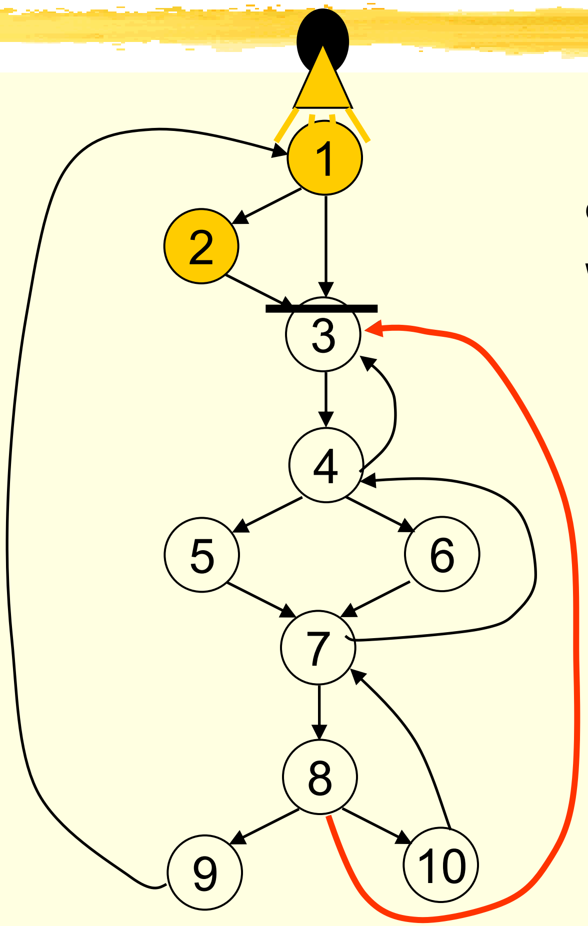
(10,7)	{7,8,10}
(7,4)	{4,5,6,7,8,10}
(8,3)	

An Example



Find all back edges in this graph and the natural loop associated with each back edge

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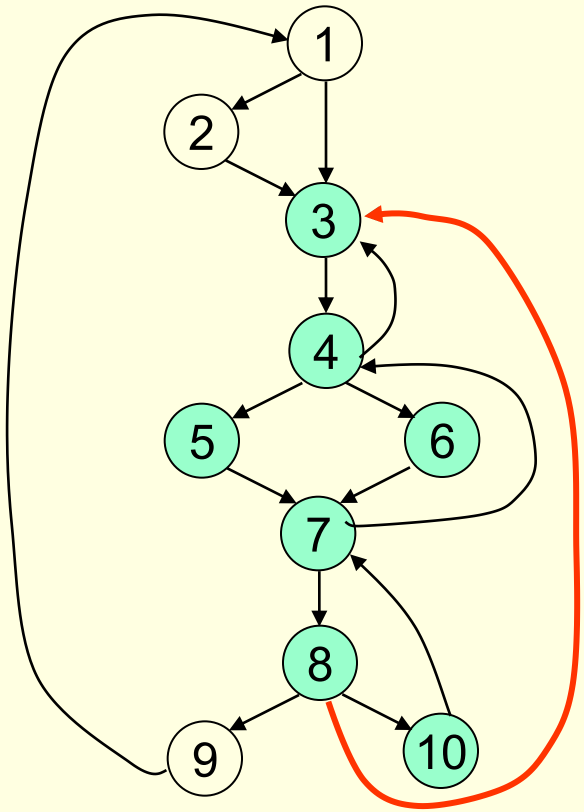
(9,1)	Entire graph
(10,7)	{7,8,10}
(7,4)	{4,5,6,7,8,10}
(8,3)	

An Example



Find all back edges in this graph and the natural loop associated with each back edge

Back edge	Natural loop
(9,1)	Entire graph
(10,7)	{7,8,10}
(7,4)	{4,5,6,7,8,10}
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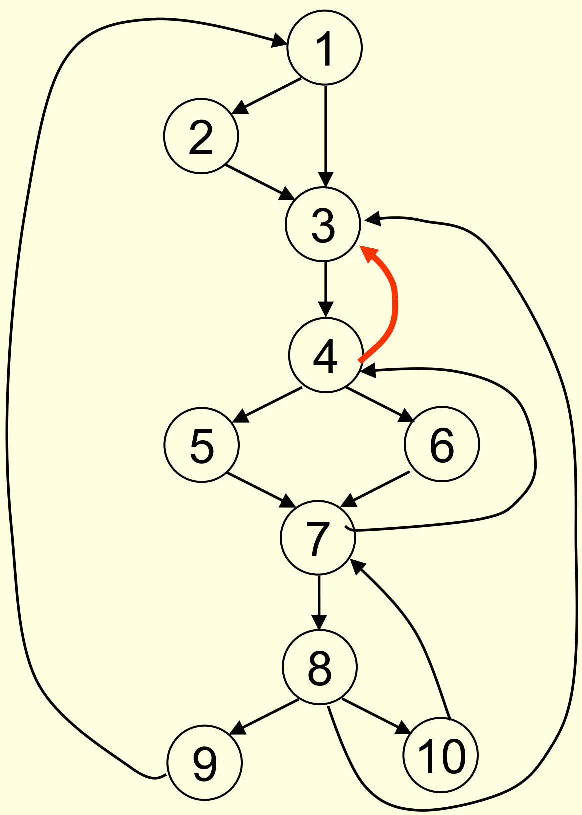
An Example



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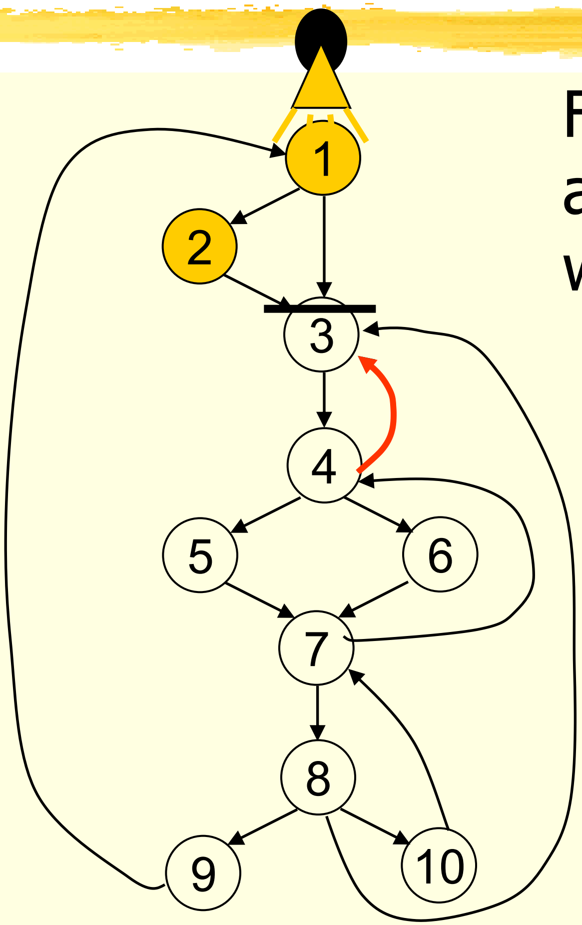

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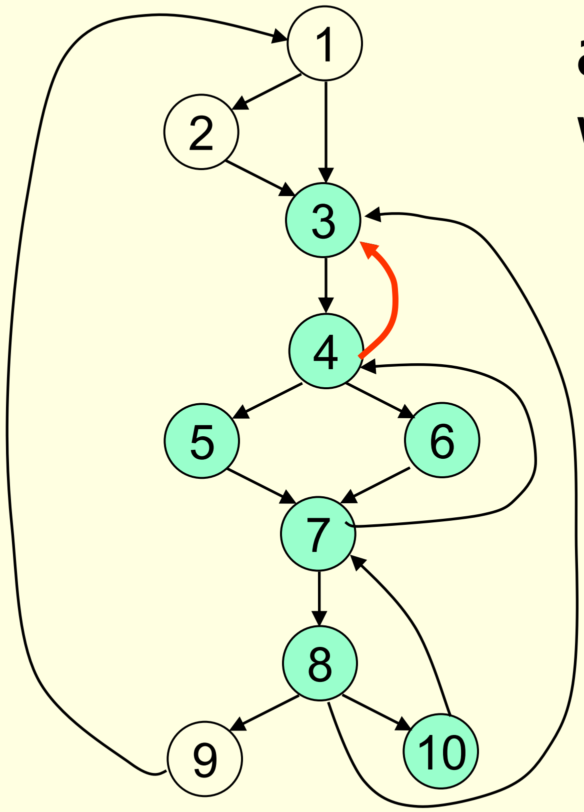

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Reducible Flow Graphs

Def: a *CFG = {V, E}* is reducible iff *E* can be partitioned into two classes:

1. **Forward edges:** form an acyclic graph where every node is reachable from the initial node
2. **Back edges**

Motivation: Structured programs always "***reducible***"
(*note: programs with gotos are still often reducible*)

Intuition: No jumps into the middle of loops.

How to check if a graph G is reducible?

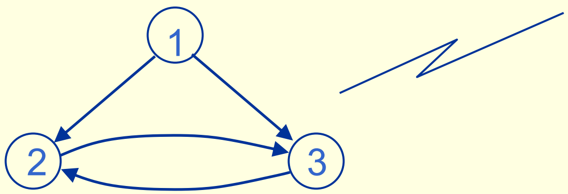
Step1: compute "dom" relation

Step2: identify all back edges

Step3: remove all back edges and derive *G'*

Step4: check if *G'* is acyclic

Example:



Bad cycle: can be entered from 2 different places

Loops in Reducible Flow Graphs

Intuitive: no bad loops.

In fact, all loops in structured programs are natural loops

In practice:

Structured programs only produce reducible graphs.