

Topic 4b – Program Execution Models

Dataflow Model of Computation: A Fine-Grain Kind of PXM

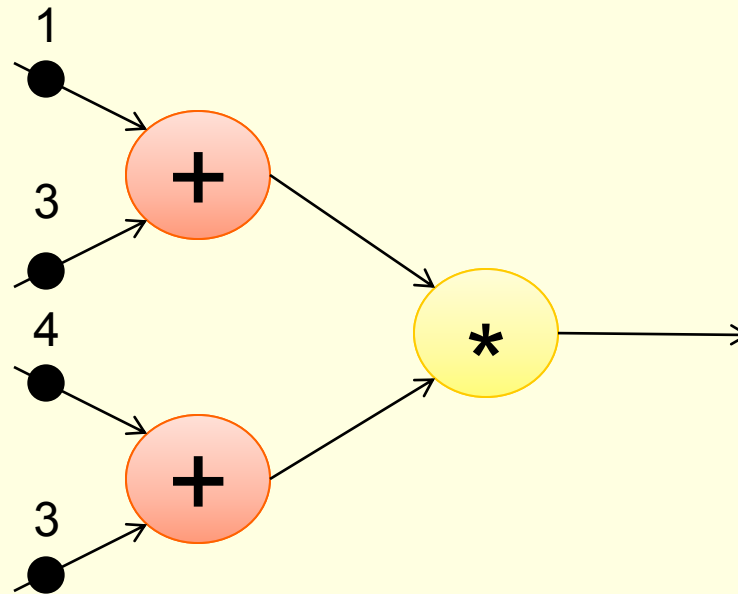


CPEG421/621: Compiler Design

Most slides taken from Prof. Gao's and J.Manzano's previous courses, with additional material from J.Landwehr.

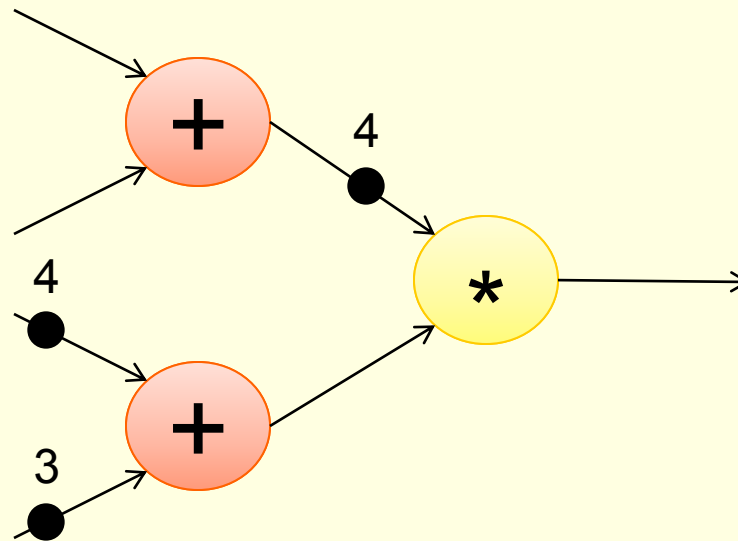
Dataflow Model of Computation

a b c d e



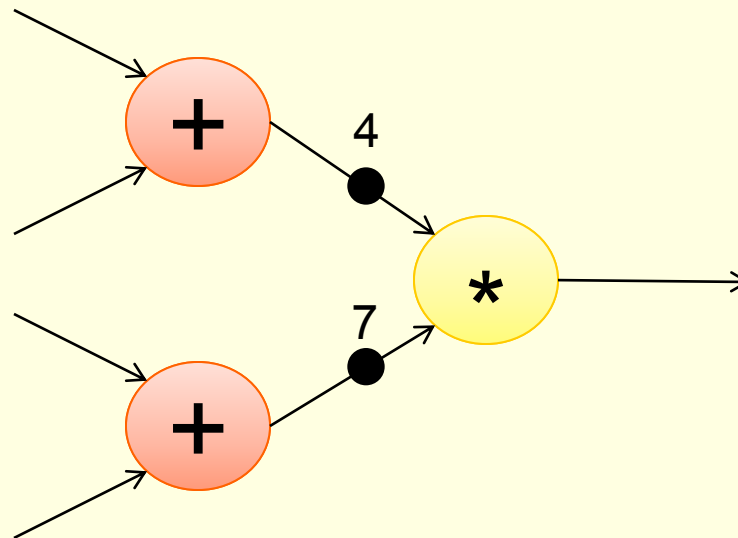
Dataflow Model of Computation

a b c d e



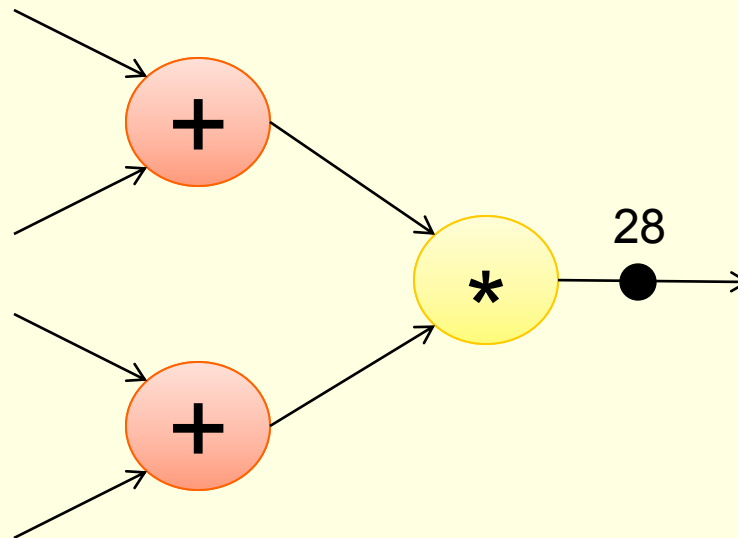
Dataflow Model of Computation

a b c d e



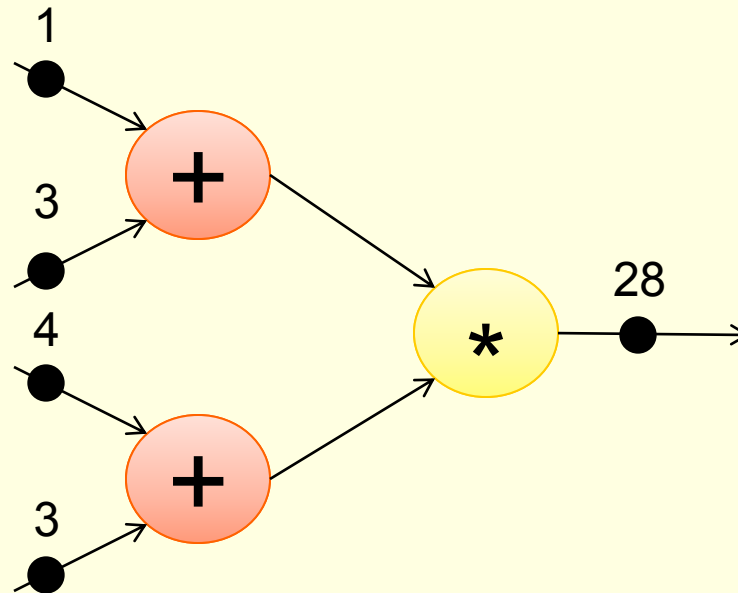
Dataflow Model of Computation

a b c d e



Dataflow Model of Computation

a b c d e



Dataflow Software Pipelining

A Base-Language



~ Data Flow Graphs ~

To serve as an intermediate-level
language for high-level languages

To serve as a machine language for
parallel machines

- J.B. Dennis

Data Flow Years at MIT

1974 – 1975

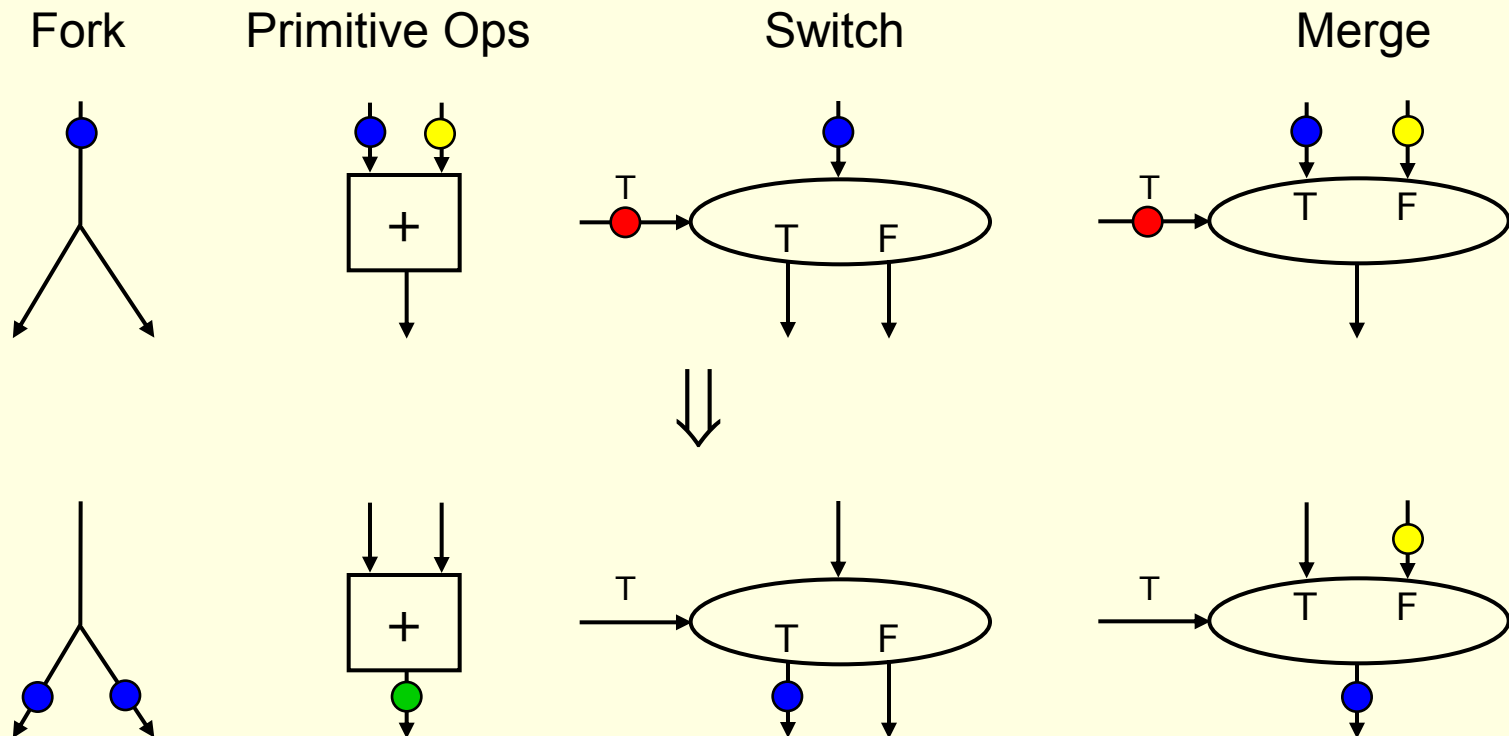
- □ April 1974: Symposium on Programming, Paris. Dennis: “*First Version of a Data Flow Procedure Language*”.
- □ January 1975: Second Annual Symposium on Computer Architecture, Houston. Dennis and Misunas: “*A Preliminary Architecture for a Basic Data-Flow Processor*”.
- □ August 1975: 1975 Sagamore Computer Conference on Parallel Processing:
 - □ Rumbaugh: “*Data Flow Languages*”
 - □ Rumbaugh: “*A Data Flow Multiprocessor*”
 - □ Dennis: “*Packet Communication Architecture*”
 - □ Misunas: “*Structure Processing in a Data-Flow Computer*”

Early Roots on Dataflow Work at MIT in 70s

- ☐ Asynchronous Digital Logic [Muller, Bartky]
- ☐ Control Structures for Parallel Programming:
[Conway, McIlroy, Dijkstra]
- ☐ Abstract Models for Concurrent Systems:
[Petri, Holt]
- ☐ Theory of Program Schemes [Ivanov, Paterson]
- ☐ Structured Programming [Dijkstra, Hoare]
- ☐ Functional Programming [McCarthy, Landin]

Dataflow Operators

- A small set of dataflow operators can be used to define a general programming language



Dataflow Graphs

```
x = a + b;  
z = b * 7;  
z = (x - y) * (x + y);
```



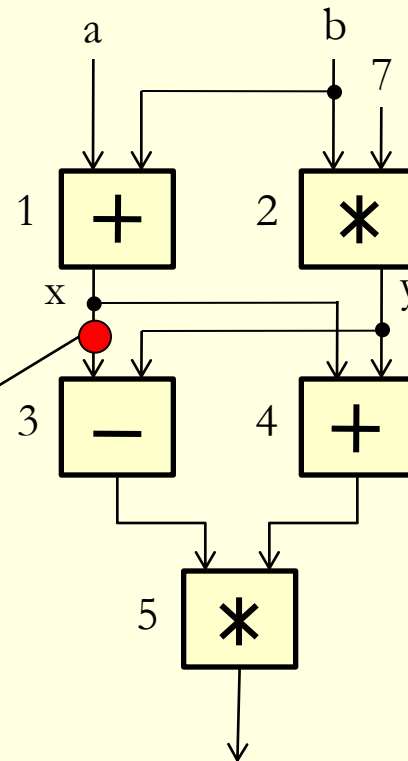
Values in dataflow graphs are represented as tokens of the form:

$\langle ip, p, v \rangle$

$\langle 3, \text{Left}, \text{value} \rangle$

Where **ip** is the instruction pointer **p** is the port and **v** represents the data

An operator executes when all its input tokens are present; copies of the result token are distributed to the destination operators.



No separate control flow

Operational Semantics

(Firing Rule)

- Values represented by tokens
- Placing tokens on the arcs
(assignment)
 - snapshot/configuration: state
- Computation
configuration $\longrightarrow$ configuration

Operational Semantics

Firing Rule



- Tokens → Data
- Assignment → Placing a token in the output arc
- Snapshot / configuration: state
- Computation
 - The intermediate step between snapshots / configurations
- An actor of a dataflow graph is enabled if there is a token on each of its input arcs

Operational Semantics

Firing Rule



- Any enabled actor may be fired to define the “next state” of the computation
- An actor is fired by removing a token from each of its input arcs and placing tokens on each of its output arcs.
- Computation → A Sequence of Snapshots
 - Many possible sequences as long as firing rules are obeyed
 - Determinacy
 - “Locality of effect”

General Firing Rules



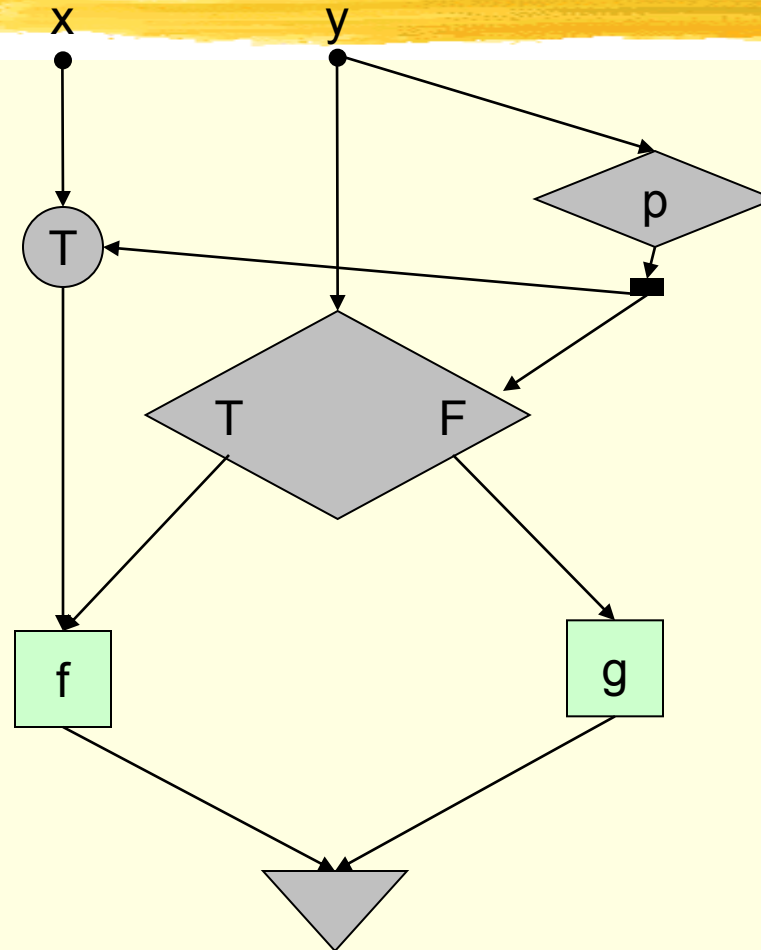
- A switch actor is enabled if a token is available on its control input arc, as well as the corresponding data input arc.

The firing of a switch actor will remove the input tokens and deliver the input data value as an output token on the corresponding output arc.
- A (unconditional) merge actor is enabled if there is a token available on any of its input arcs.

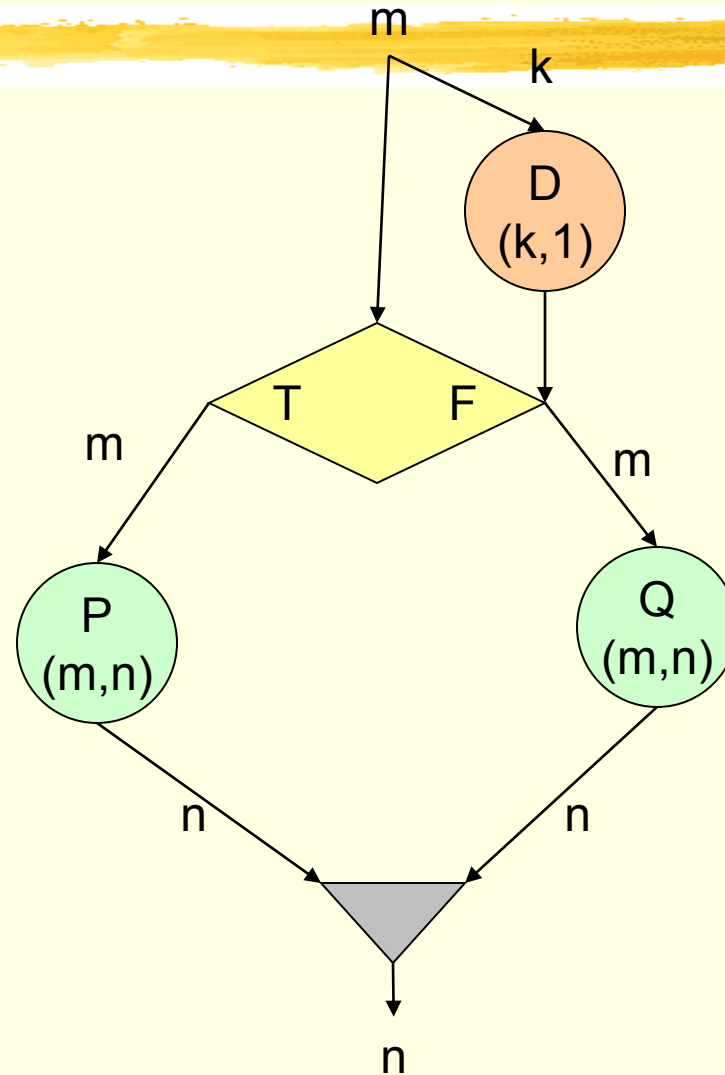
An enabled (unconditional) merge actor may be fired and will (non-deterministically) put one of the input tokens on the output arc.

Conditional Expression

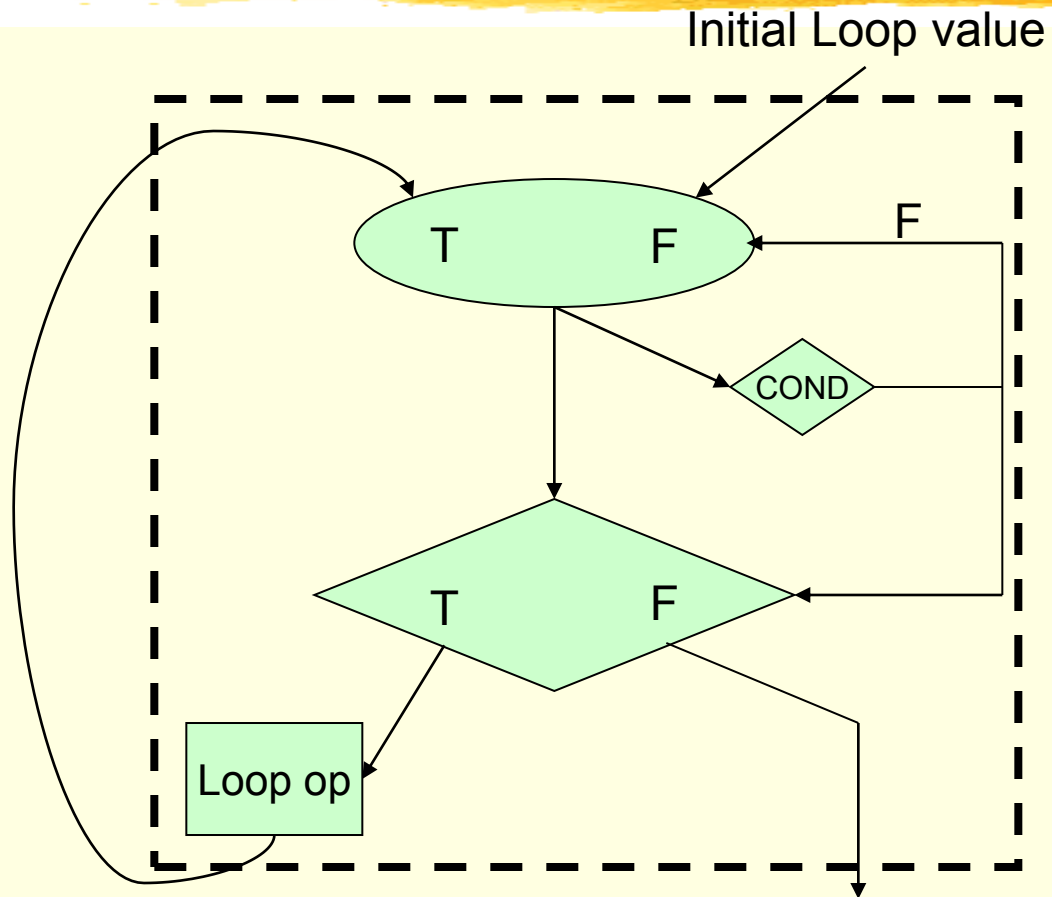
```
if (p(y)){  
    f(x,y);  
}  
else{  
    g(y);  
}
```



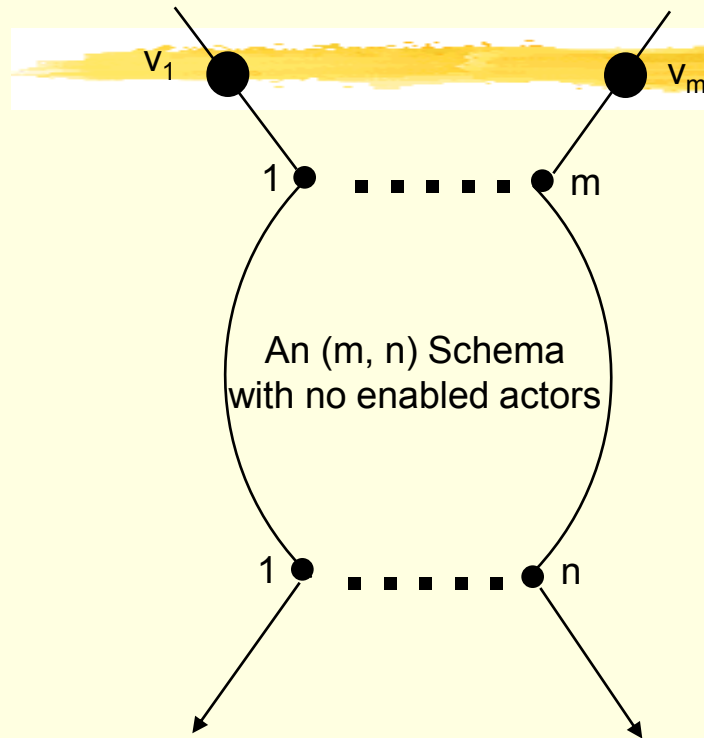
A Conditional Schema



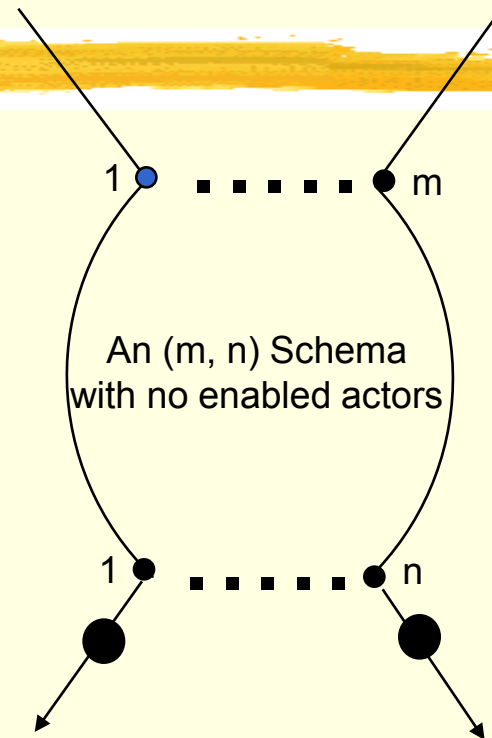
A Loop Schema



Properties of Well-Behaved Dataflow Schemata



(a) Initial Snapshot



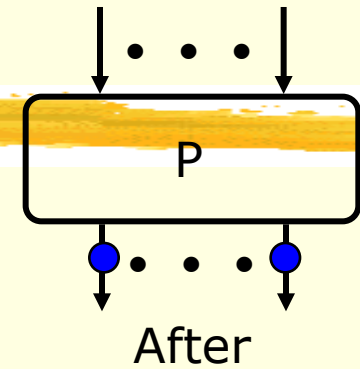
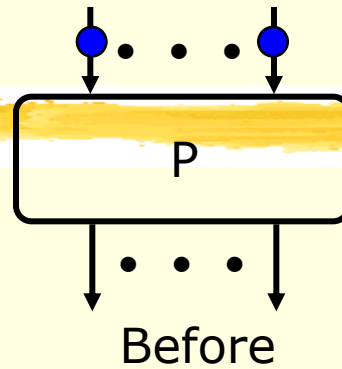
(a) Final Snapshot

Well-behaved Data Flow Graphs

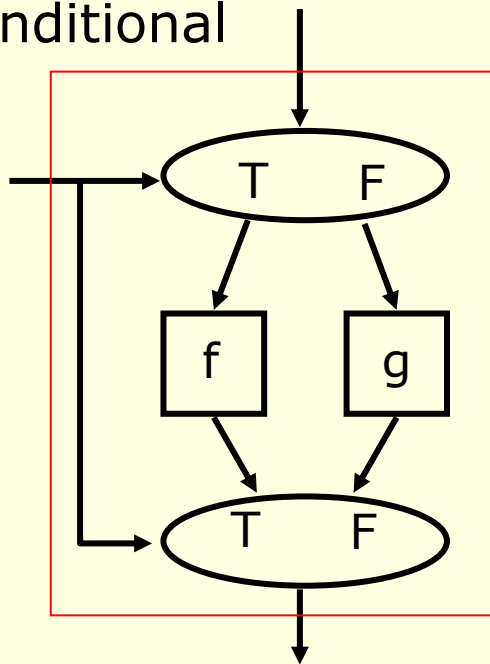
- Data flow graphs that produce exactly one set of result values at each output arcs for each set of values presented at the input arcs
- Implies the initial configuration is re-established
- Also implies *determinacy*

Well Behaved Schemas

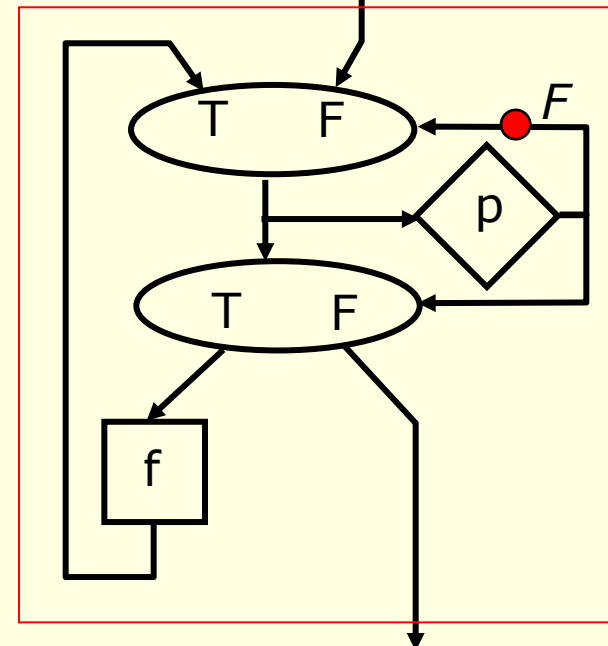
one-in-one-out
& self cleaning



Conditional



Loop



Well-formed Dataflow Schema

(Dennis & Fossen 1973)

- An operator is a WFDS
- A conditional schema is a WFDS
- A iterative (loop) schema is a WFDS
- An acyclic composition of component WFDS is a WFDS

Theorem

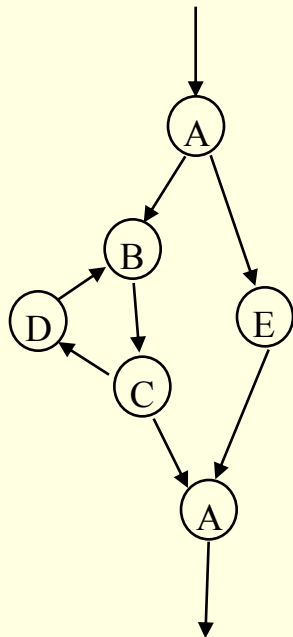


“A well-formed data flow graph is well-behaved”

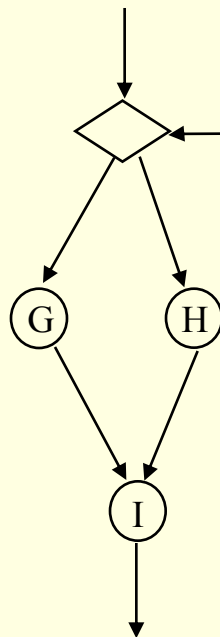
proof by induction

Example of “Sick” Dataflow Graphs

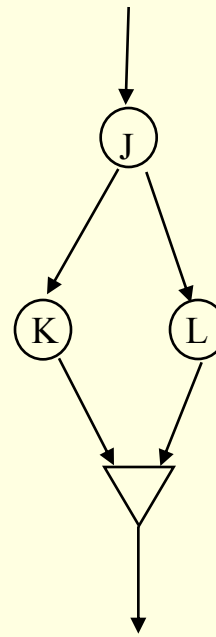
Arbitrary connections of data flow operators can result in pathological programs, such as the following:



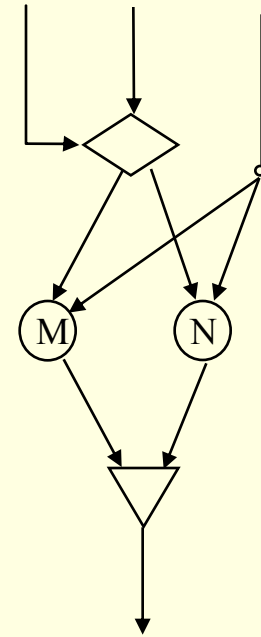
1. Deadlock



2. Hangup



3. Conflict



4. Unclean

Well-behaved Program



- Always determinate in the sense that a unique set of output values is determined by a set of input values
- References:

Rodriquez, J.E. 1966, "A Graph Model of Parallel Computation", MIT, TR-64]

Patil, S. "Closure Properties of Interconnections of Determinate Systems", Records of the project MAC conf. on concurrent systems and parallel Computation, ACM, 1970, pp 107-116]

Denning, P.J. "On the Determinacy of Schemata" pp 143-147

Karp, R.M. & Miller, R.E., "Properties of a Model of Parallel Computation Termination, termination, queuing", Appl. Math, 14(6), Nov.

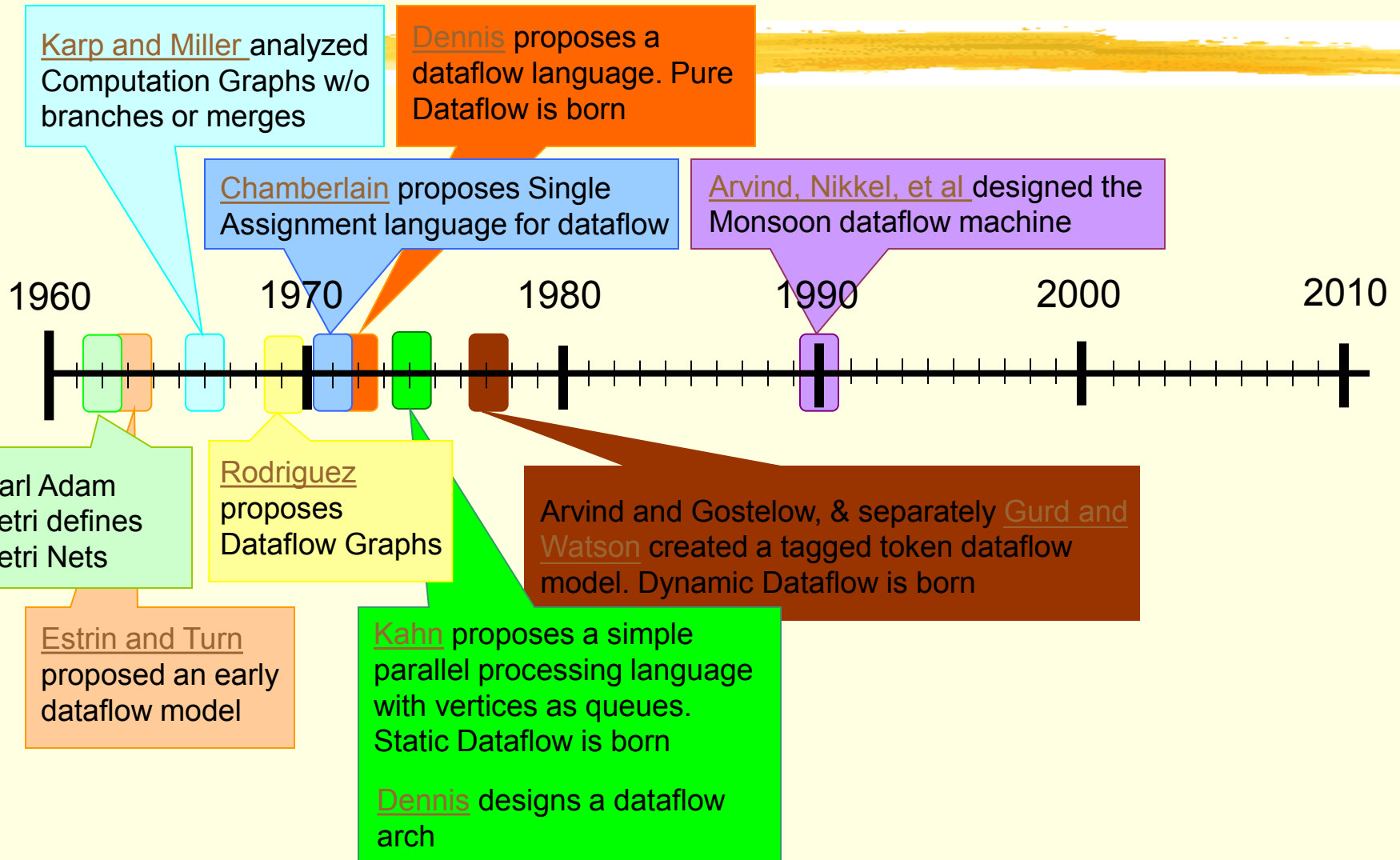
1966

Remarks on Dataflow Models



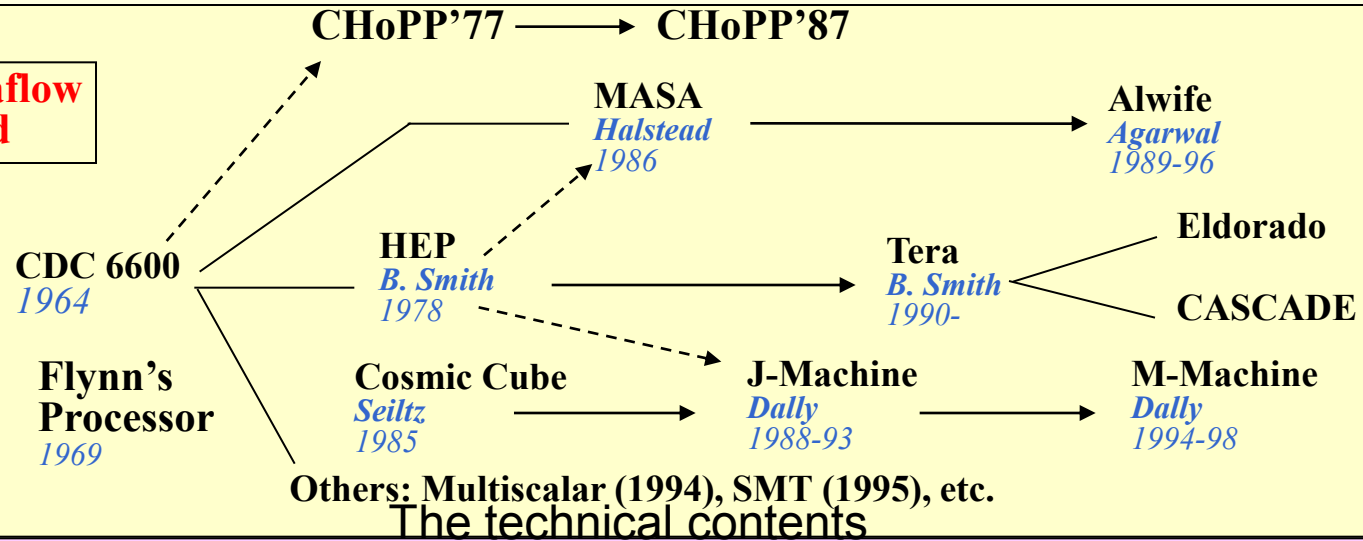
- A fundamentally sound and simple parallel model of computation (features very few other parallel models can claim)
- Few dataflow architecture projects survived passing early 1990s. But the ideas and models live on ..
- In the new multi-core age: we have many reasons to re-examine and explore the original dataflow models and learn from the past

A Short Story

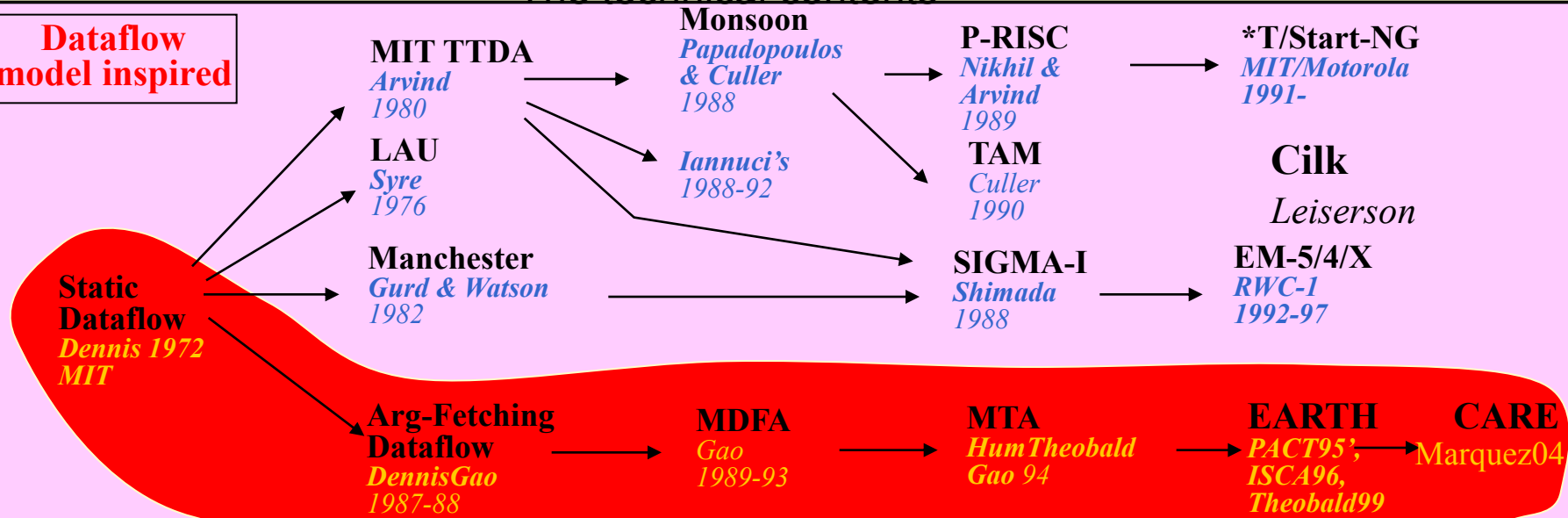


Evolution of Multithreaded Execution and Architecture Models

Non-dataflow based



Dataflow model inspired



Dataflow Models



- Static Dataflow Model
- Recursive Program Graphs
- Tagged Token Dataflow Model
Also known as *dynamic*

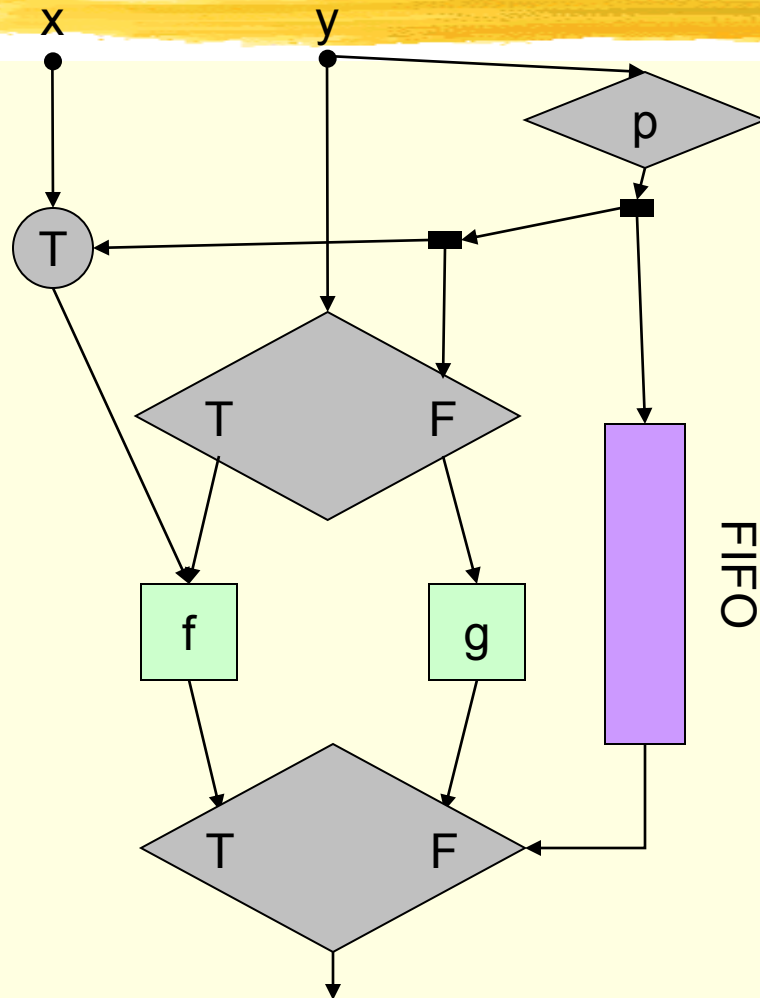
Static Dataflow Model



- “...for any actor to be enabled, there must be no tokens on any of its output arcs...”
- So-called: at most “one-token-per-arc” rule

Conditional Expression

```
if (p(y)){  
    f(x,y);  
}  
else{  
    g(y);  
}
```



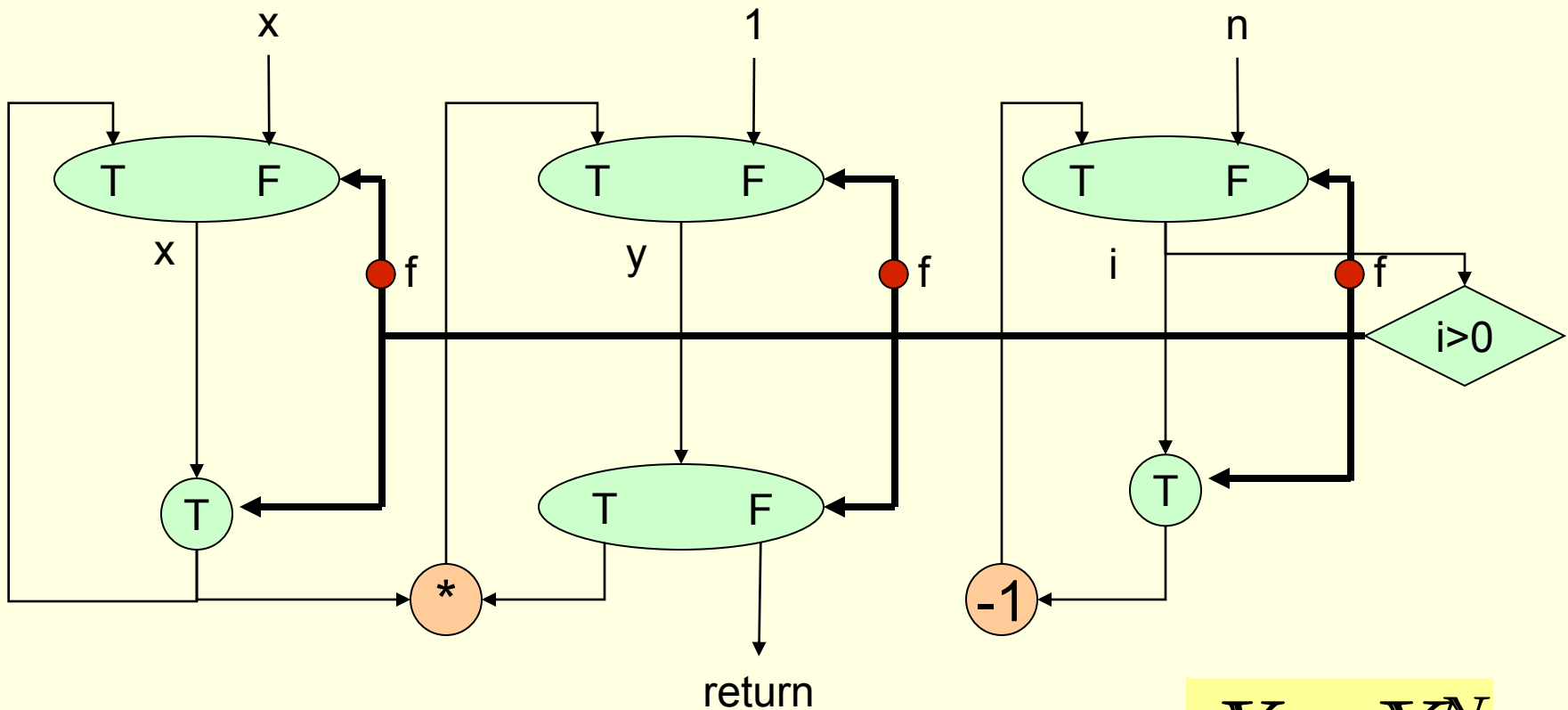
Example

Power Function

```
long power(int x, int n){  
    int y = 1;  
    for(int i = n; i > 0; --i)  
        y *= x;  
    return y;  
}
```

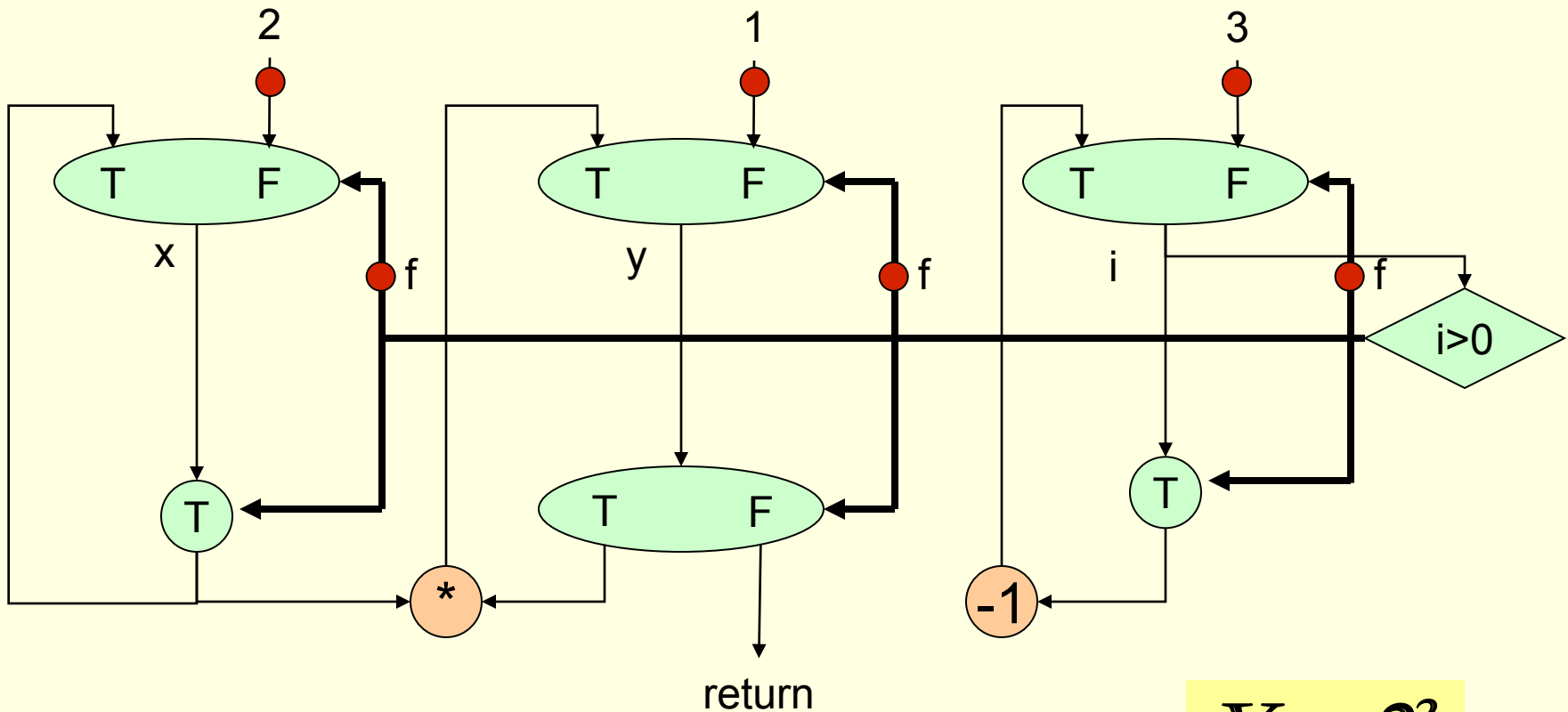
$$Y = X^N$$

Power Function



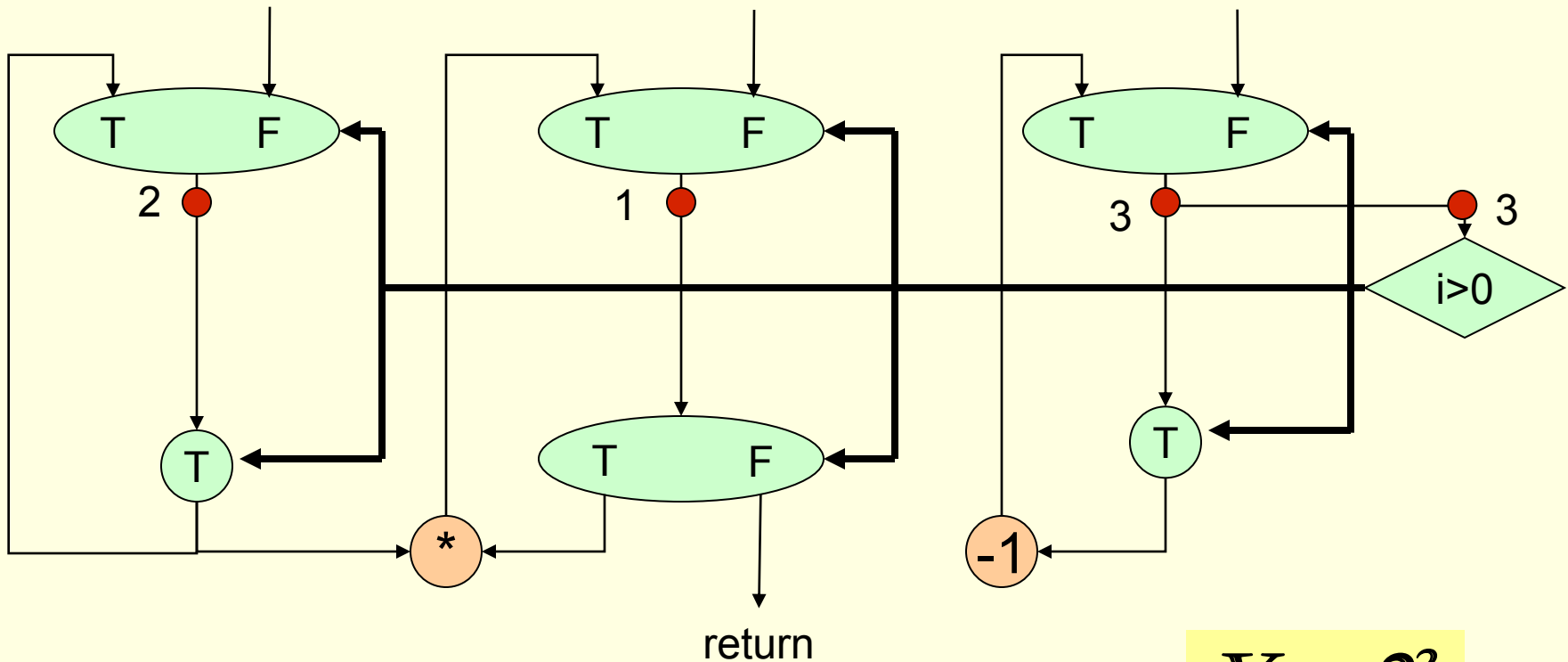
$$Y = X^N$$

Power Function



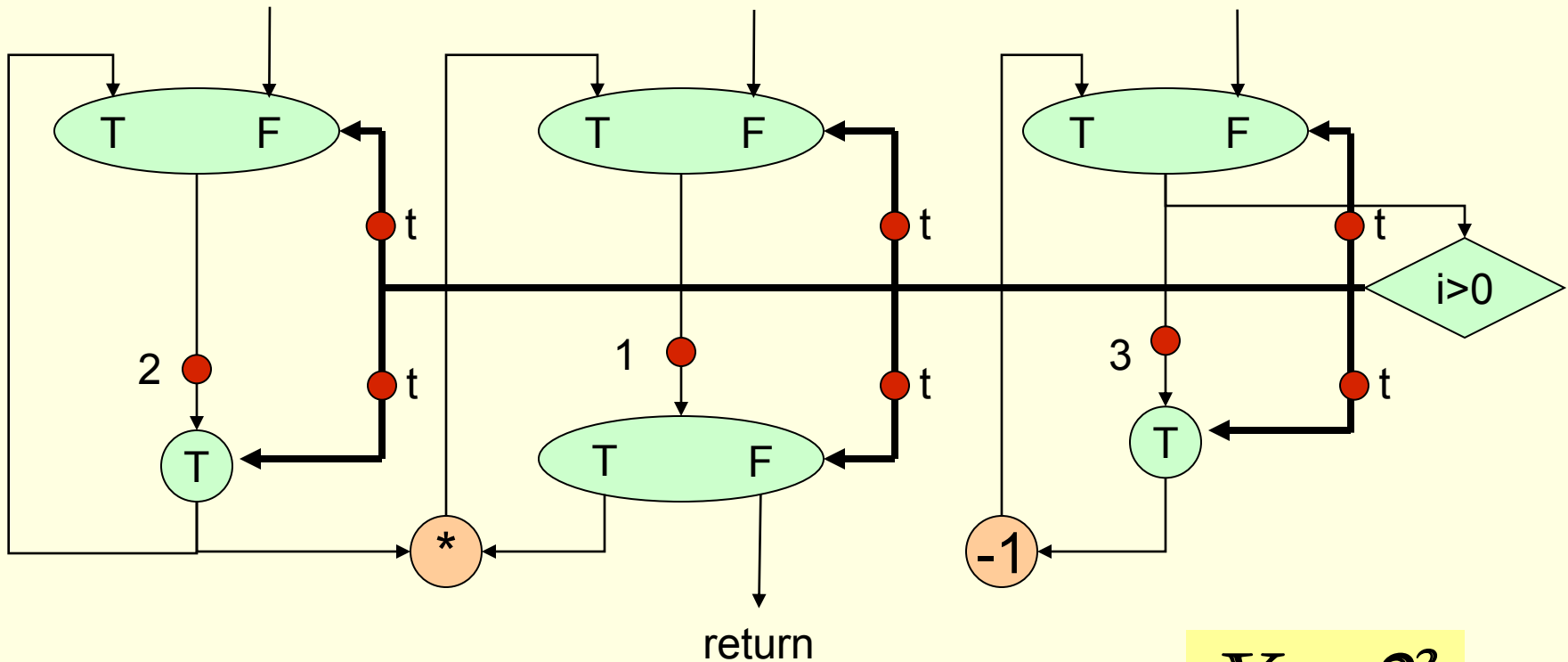
$$Y = 2^3$$

Power Function



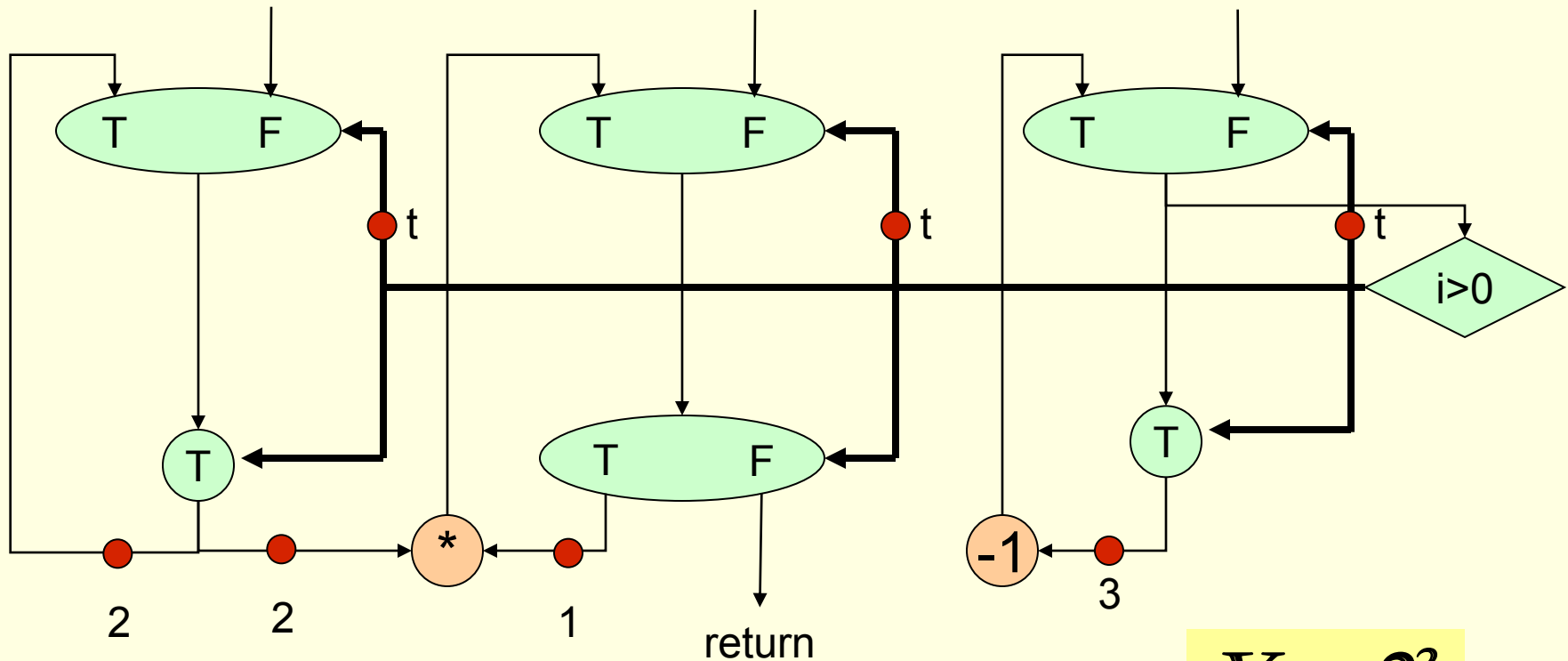
$$Y = 2^3$$

Power Function



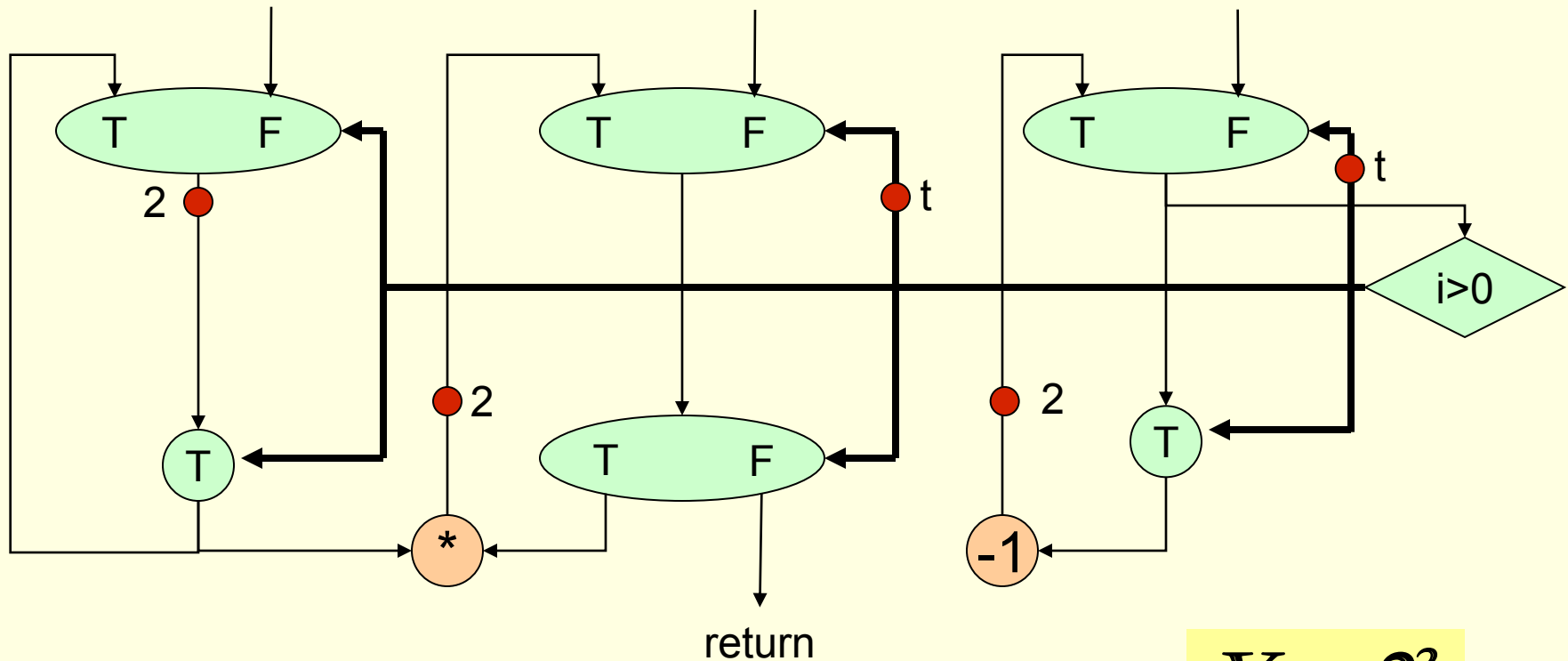
$$Y = 2^3$$

Power Function



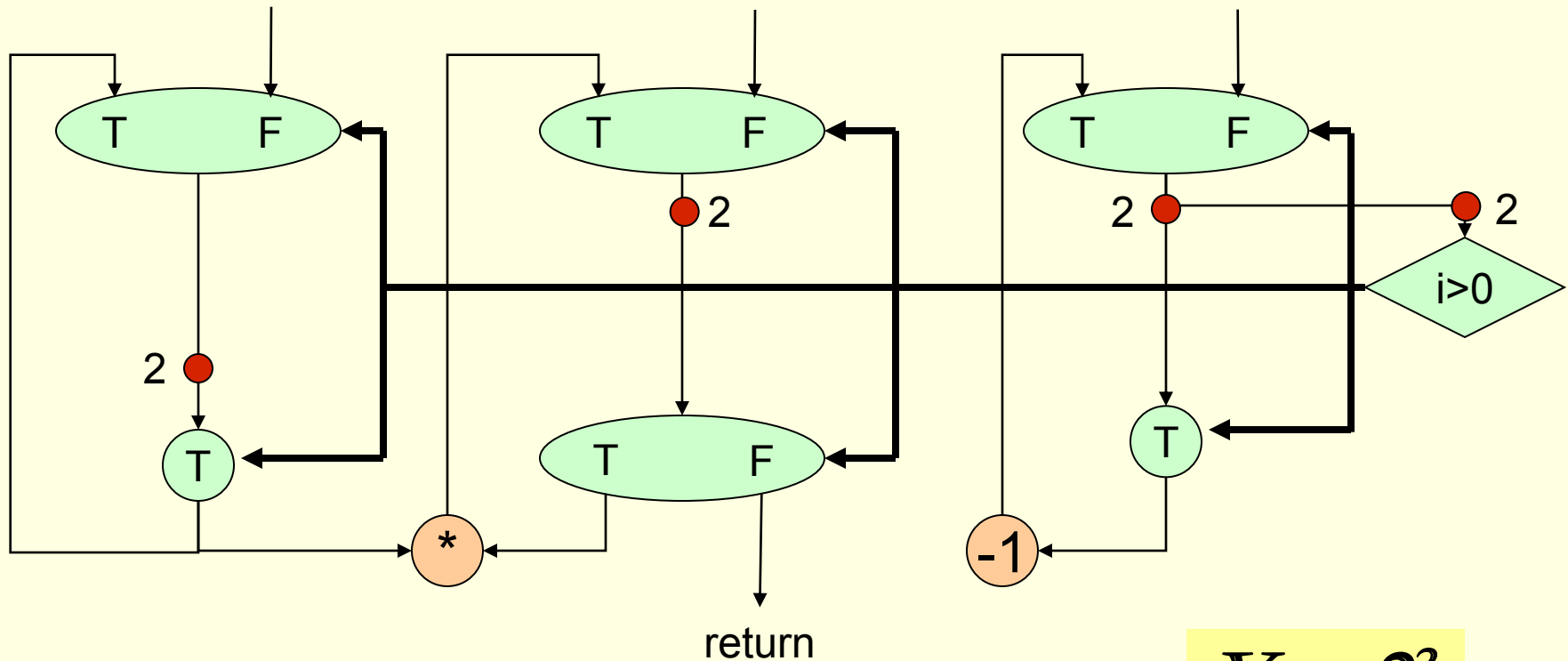
$$Y = 2^3$$

Power Function

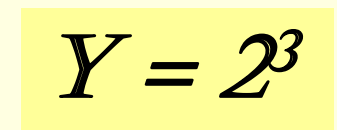


$$Y = 2^3$$

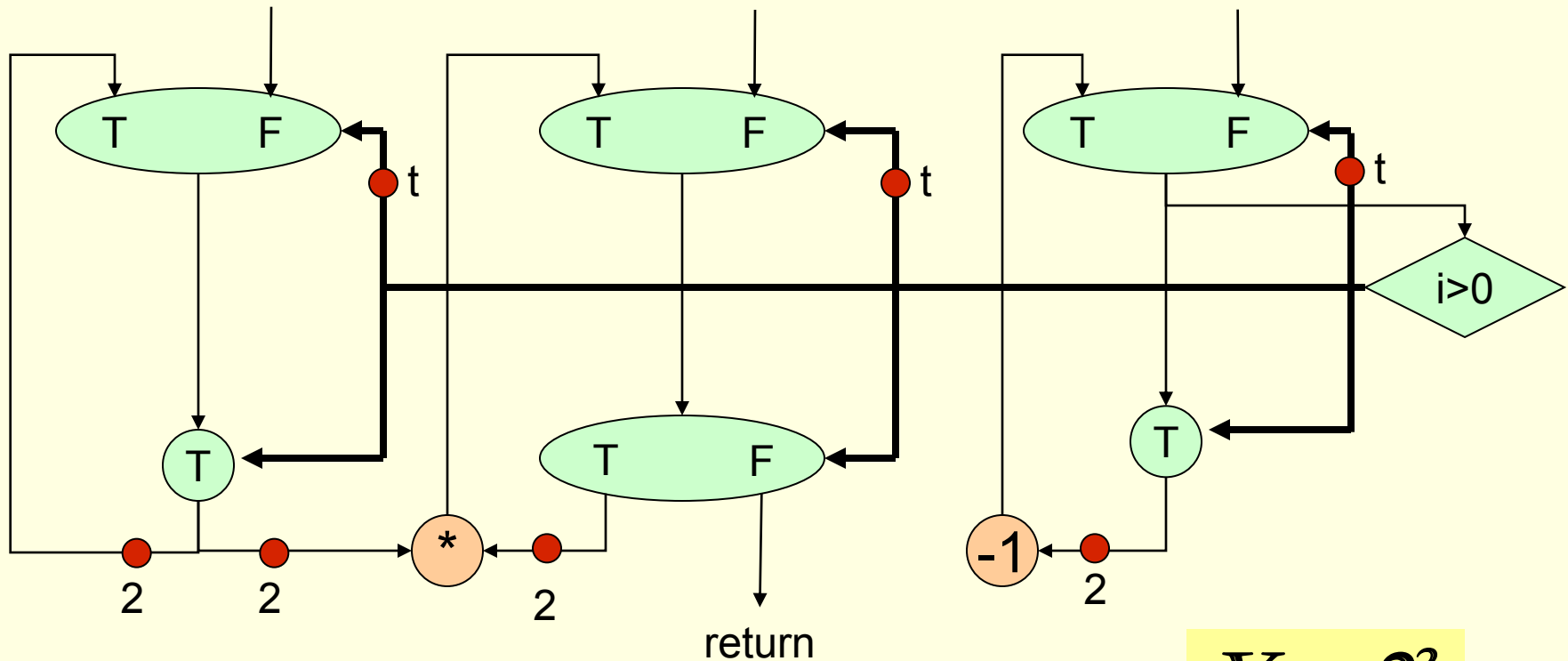
Power Function



$$Y = 2^3$$

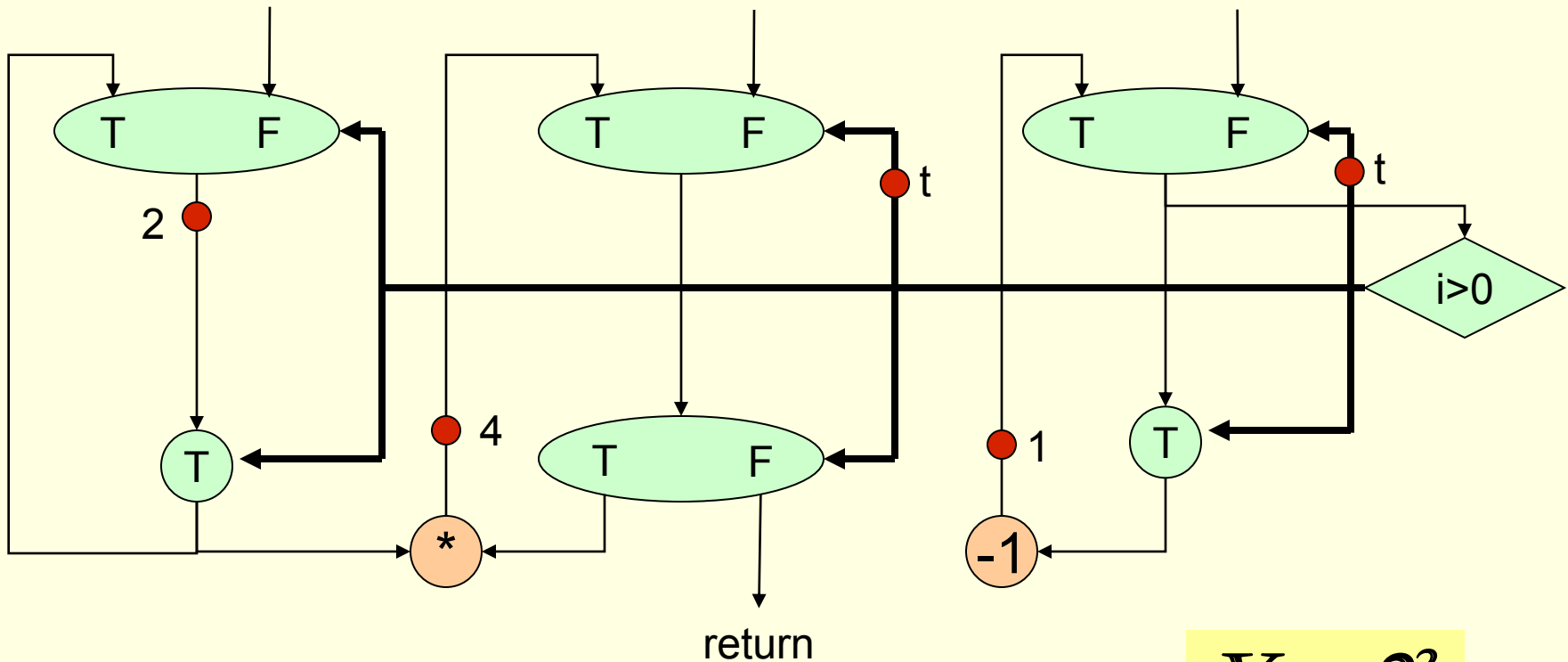


Power Function

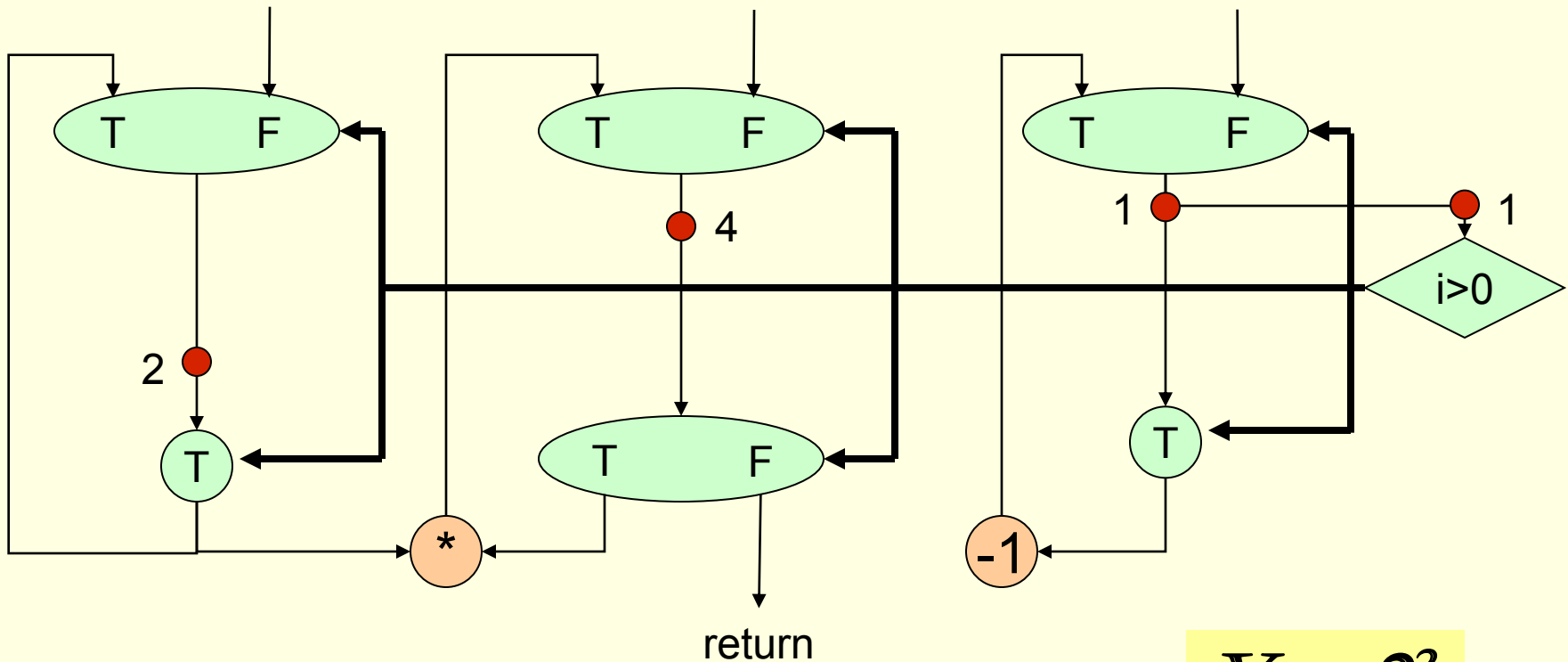


$$Y = 2^3$$

Power Function

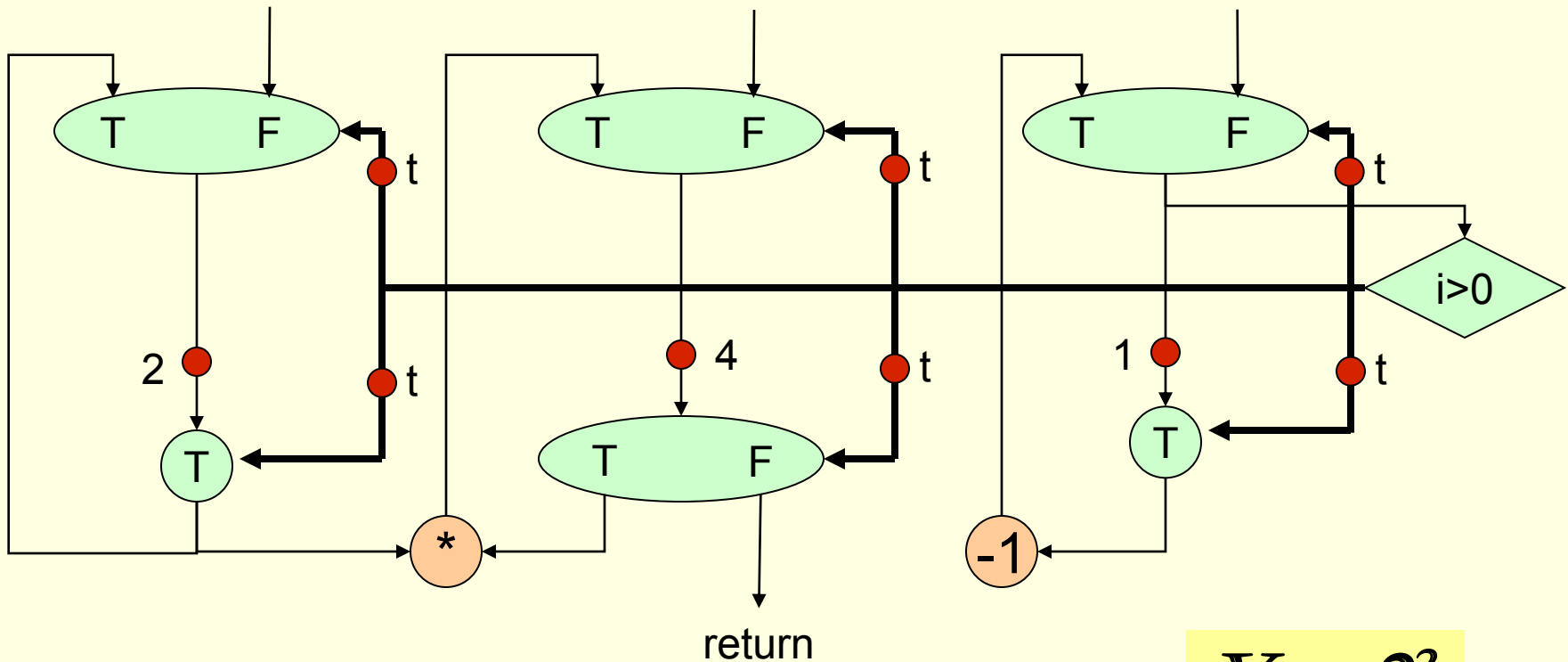


Power Function



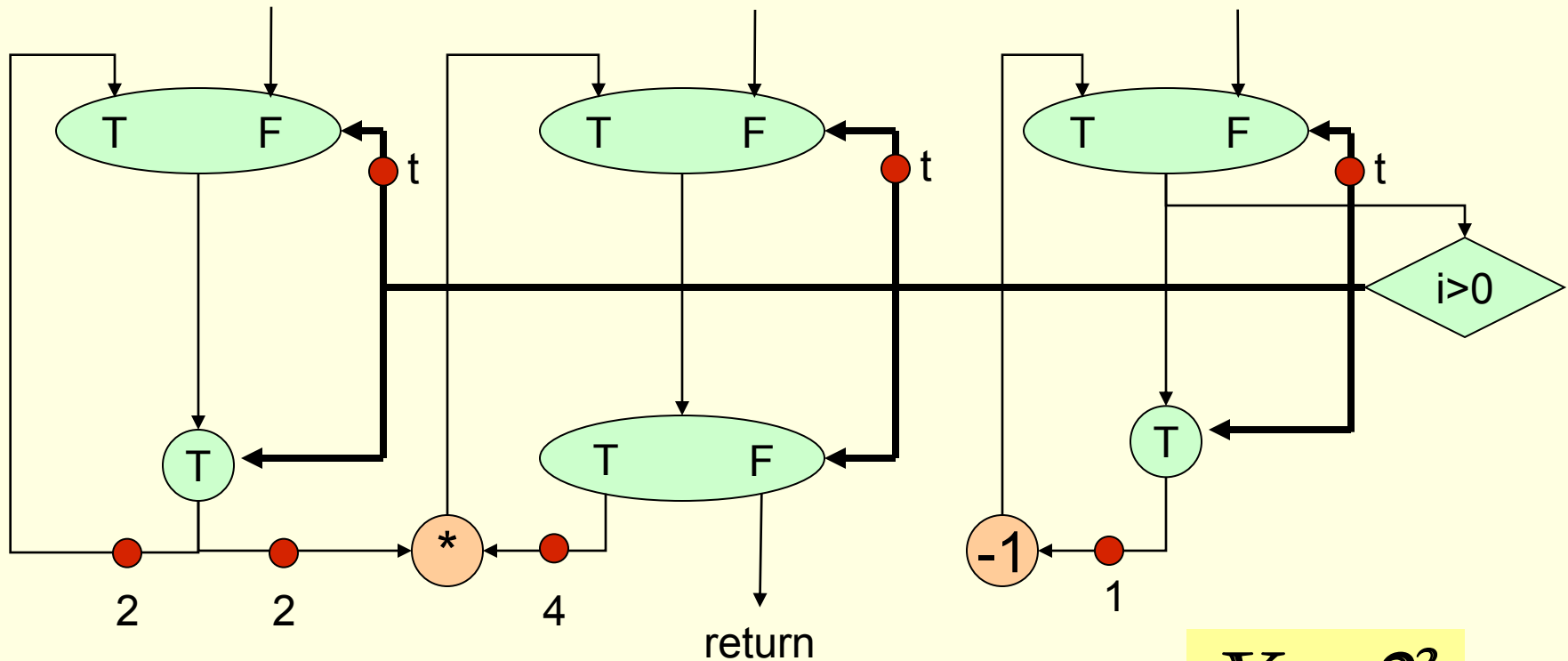
$$Y = 2^3$$

Power Function



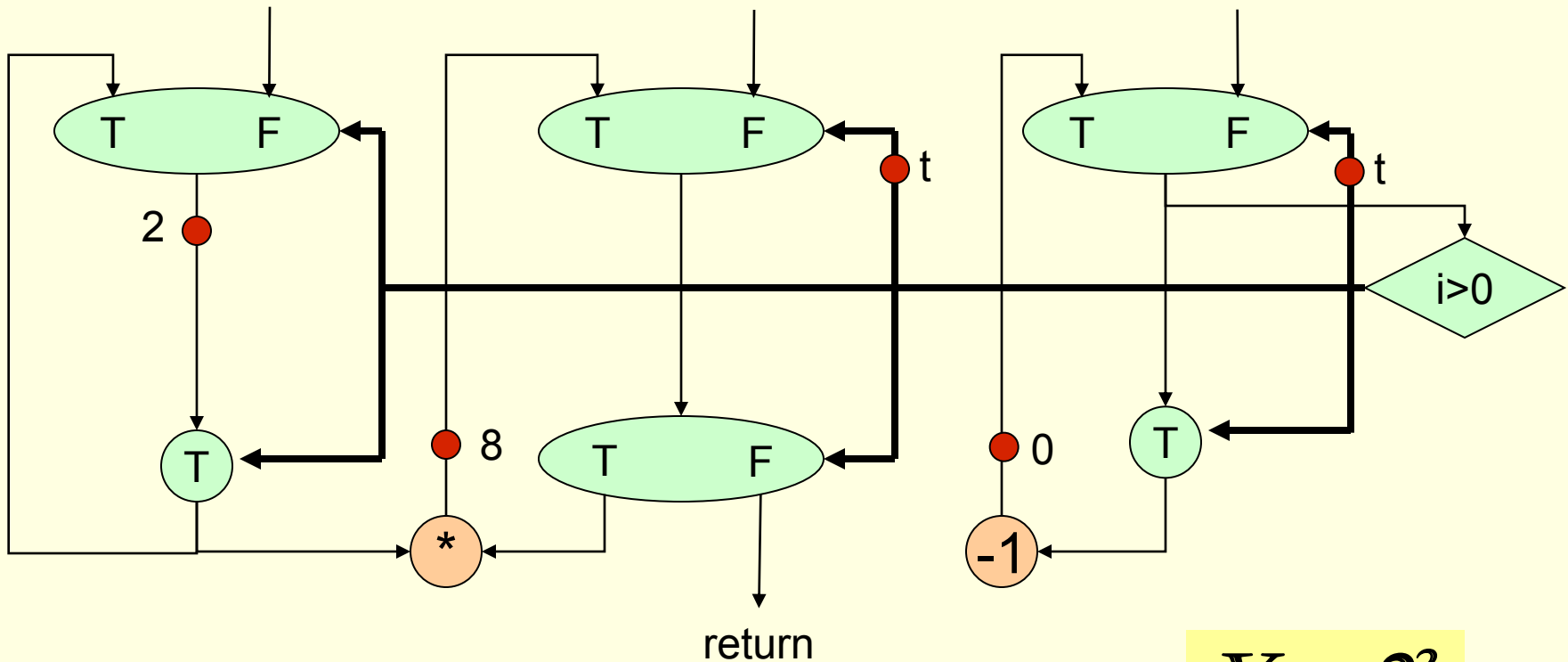
$$Y = 2^3$$

Power Function



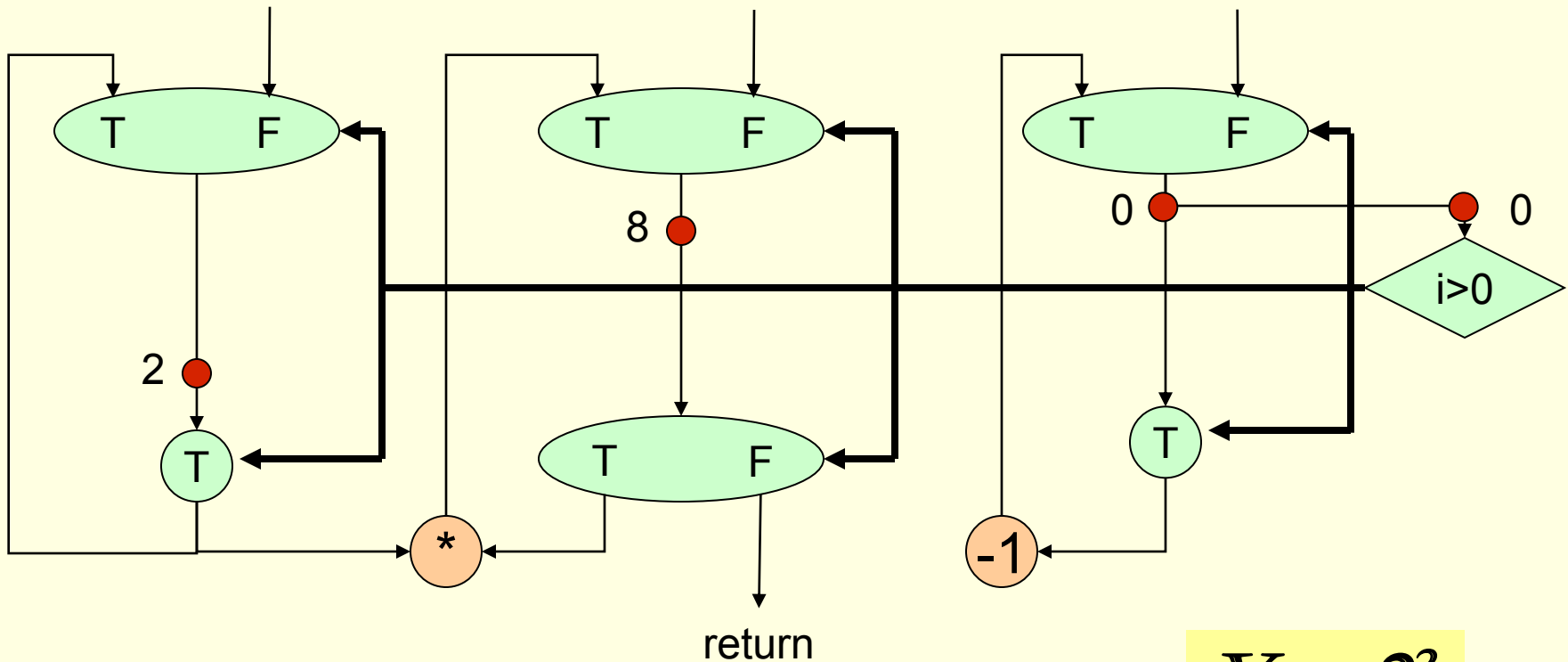
$$Y = 2^3$$

Power Function



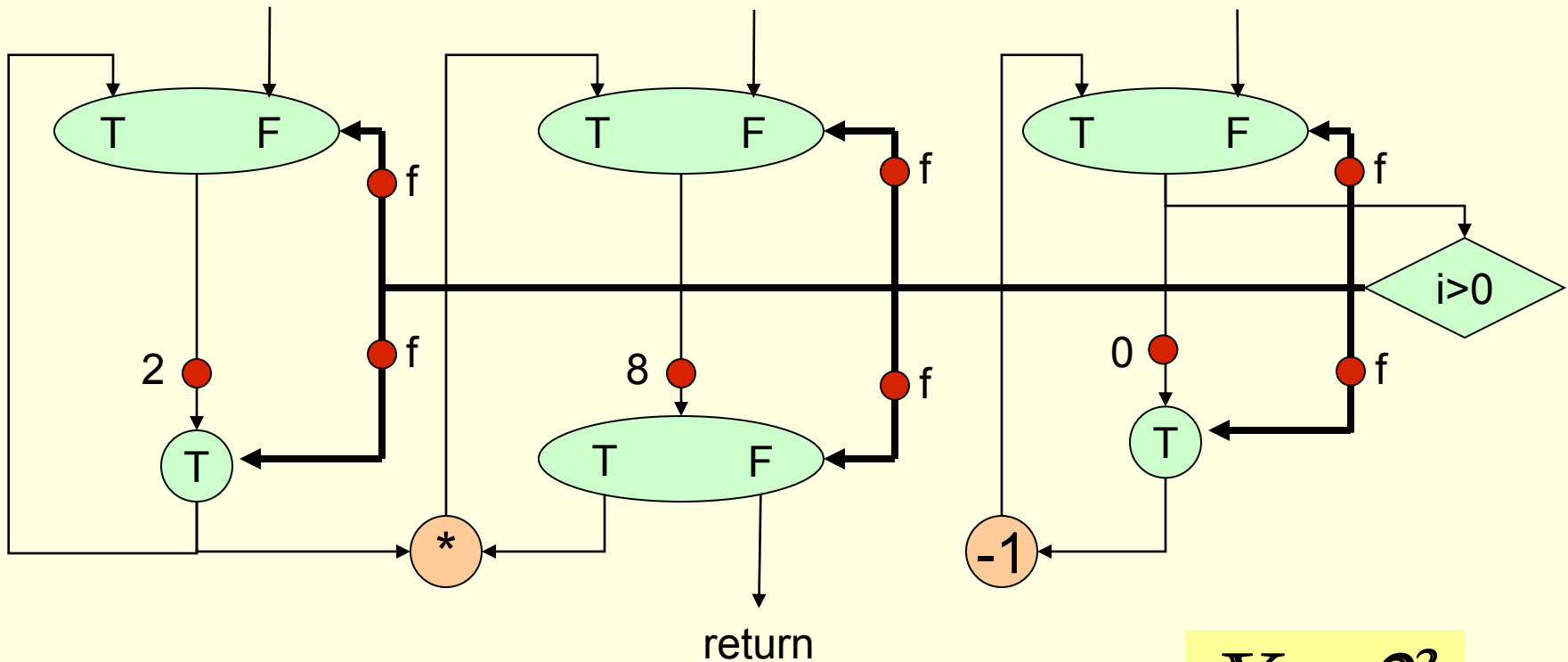
$$Y = 2^3$$

Power Function



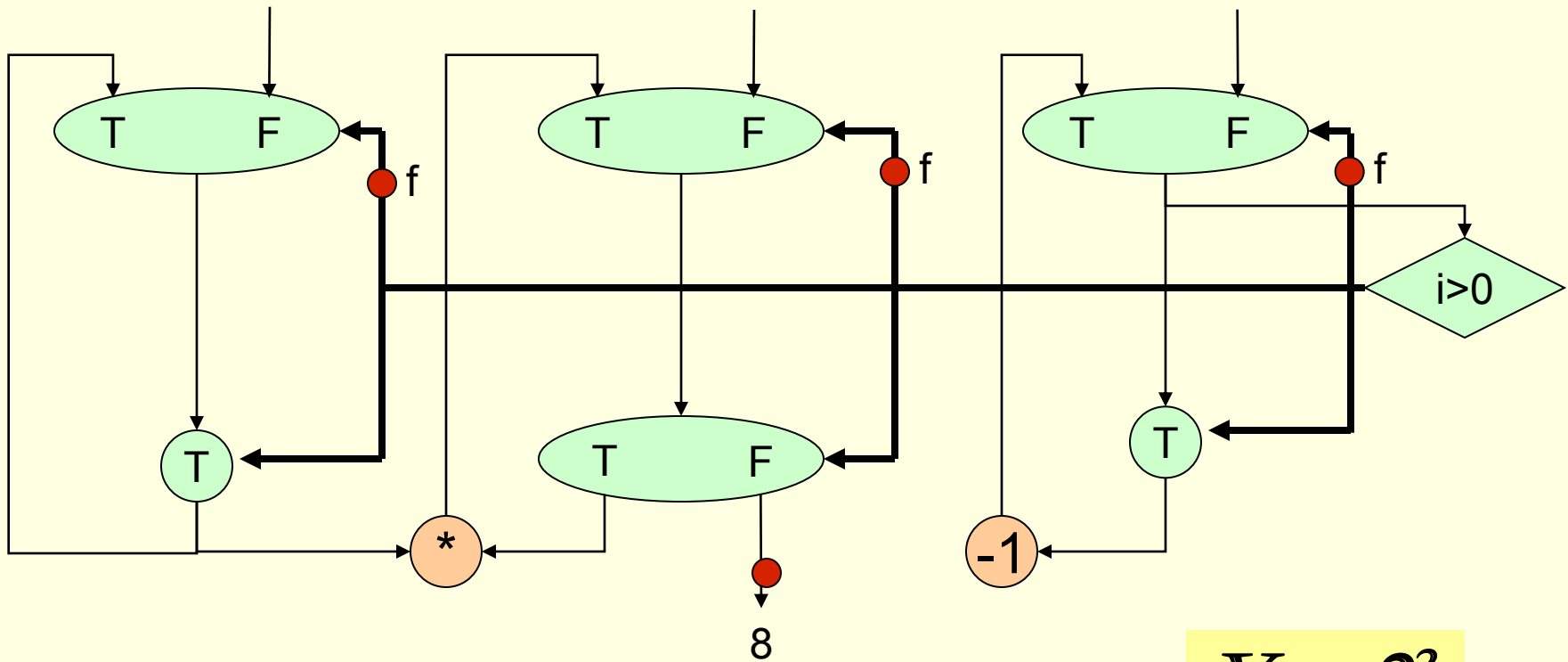
$$Y = 2^3$$

Power Function



$$Y = 2^3$$

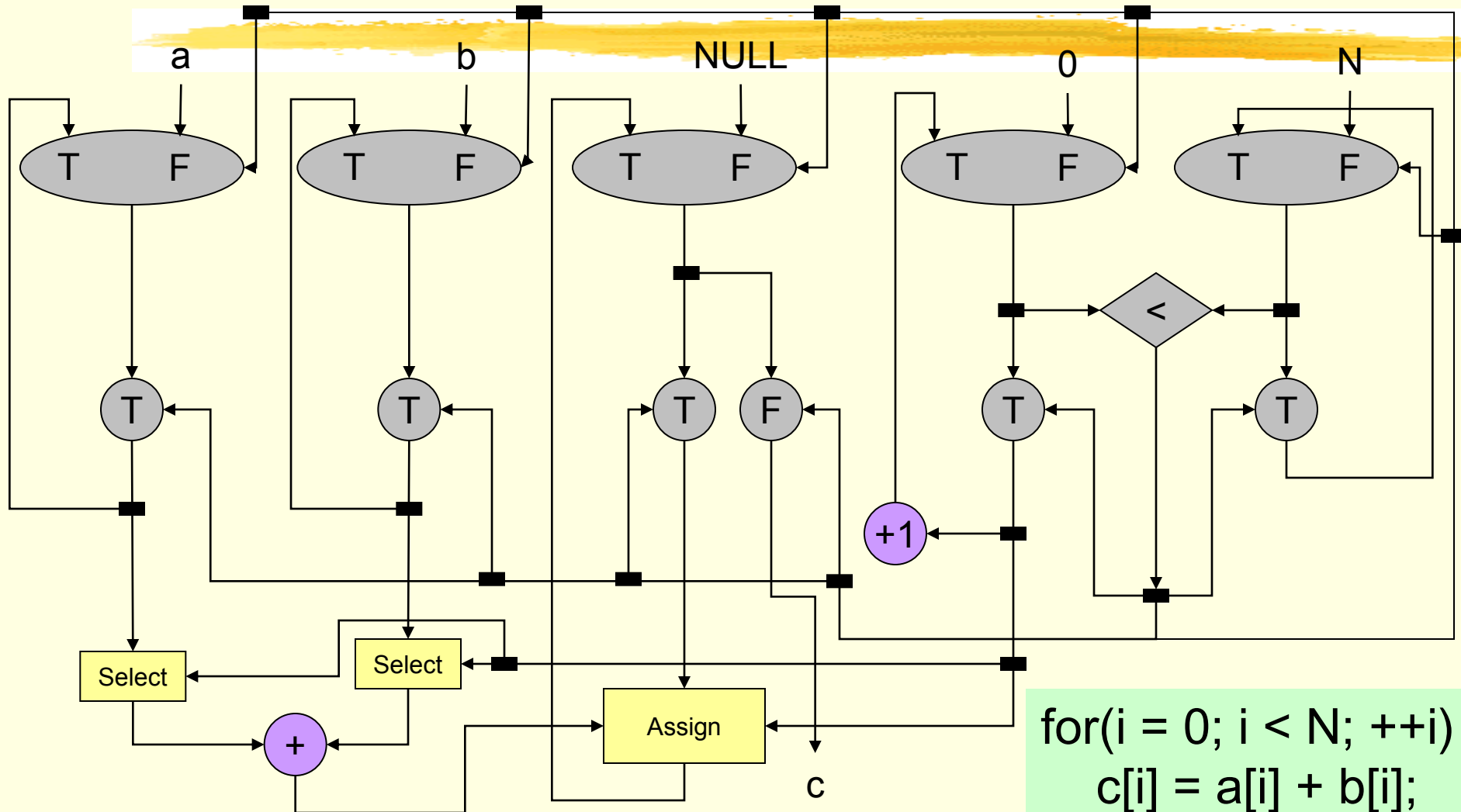
Power Function



$$Y = 2^3$$

DFG

Vector Addition



```
for(i = 0; i < N; ++i)
    c[i] = a[i] + b[i];
```

Static Dataflow Model Features

- One-token-per-arc
- Deterministic merge
- Conditional/iteration construction
- Consecutive iterations of a loop appear to be only subjected to sequential execution (?)
- A dataflow graph → activity templates
 - Opcode of the represented instruction
 - Operand slots for holding operand values
 - Destination address fields
- Token → value + destination

Static Dataflow Model Features



- Deficiencies:

Due to acknowledgment tokens, the token traffic is doubled.

Lack of support for programming constructs that are essential to modern programming language

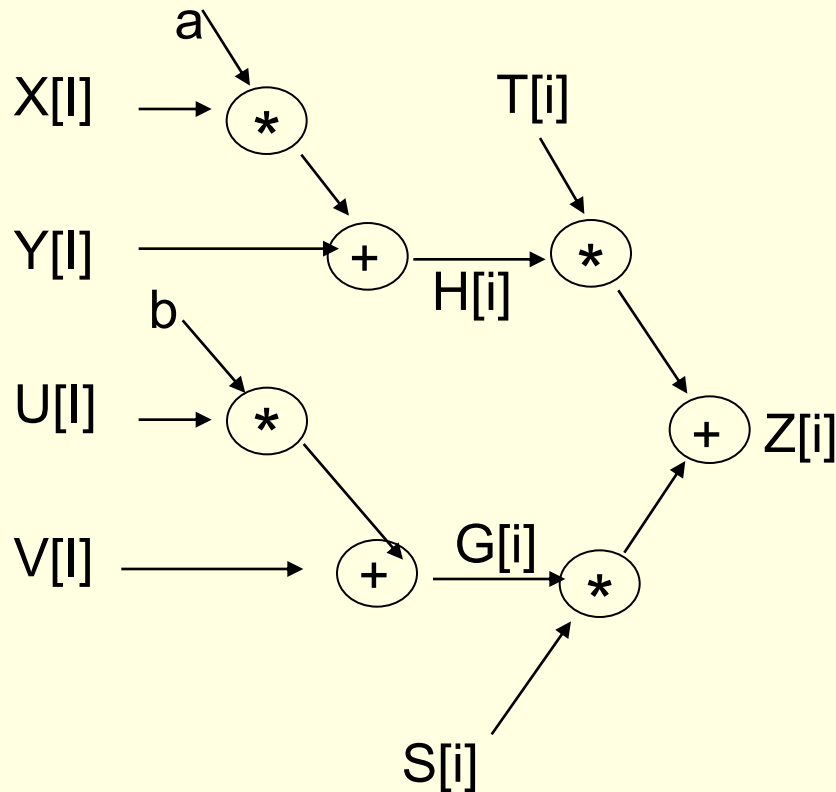
no procedure calls,

no recursion.

- Advantage:

simple model

Dataflow Software Pipelining



for I in $[1, n]$

$$H[i] = a * X[i] + Y[i]$$

$$G[i] = b * U[i] + V[i]$$

$$Z[i] = T[i] * H[i] + S[i] * G[i]$$

end for

Recursive Program Graphs



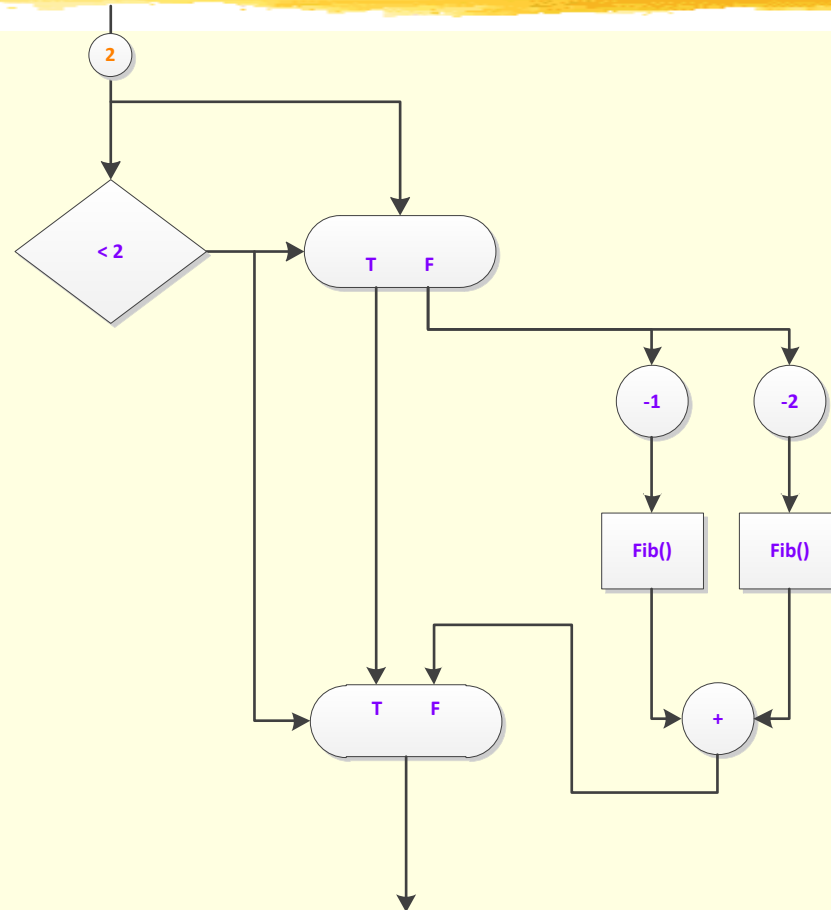
- Outlaw iterations :
Graph must be acyclic
- One-token-per-arc-per-invocation
- Iteration is expressed in terms of a tail recursion

Tail Function Application



- Tail-procedure application
a procedure application that occurs as the last statement in another procedure;
- Tail-function application is a function application (appears in the body expression) whose result value is also returned as the value of the entire functions
- Consider the role of stack

An Example with Fibonacci (Thanks to J.Landwehr)

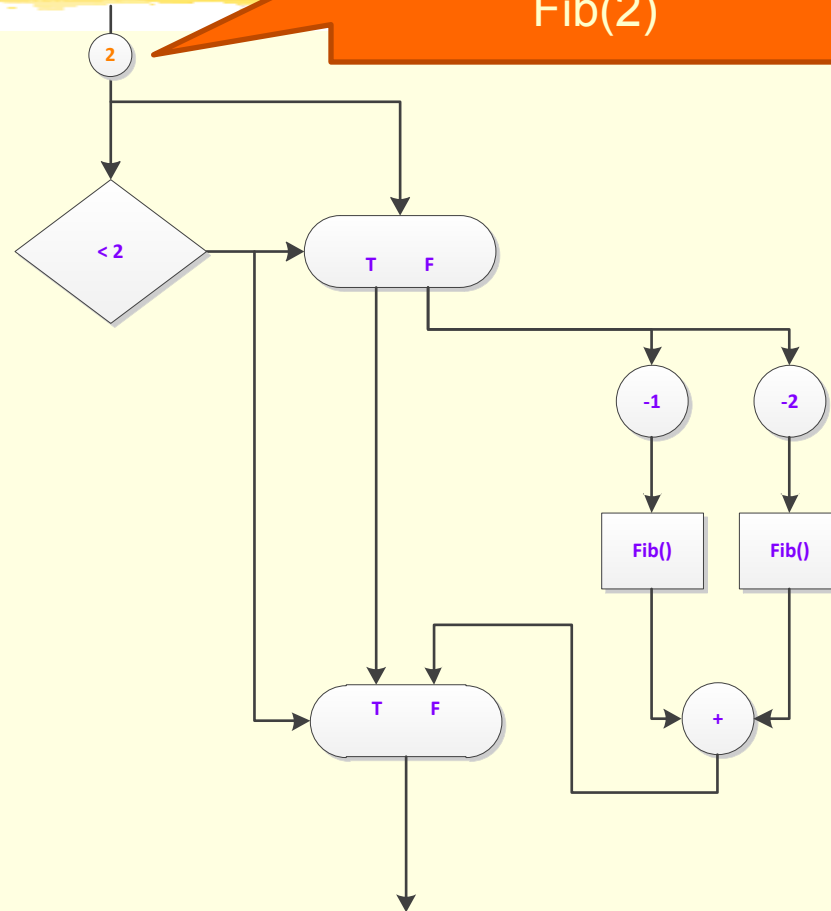


Dataflow is inherently parallel, but if it is recursively called it can lead to deadlocks if the parent is not clean

An Example with Fibonacci

(Thanks to J.La)

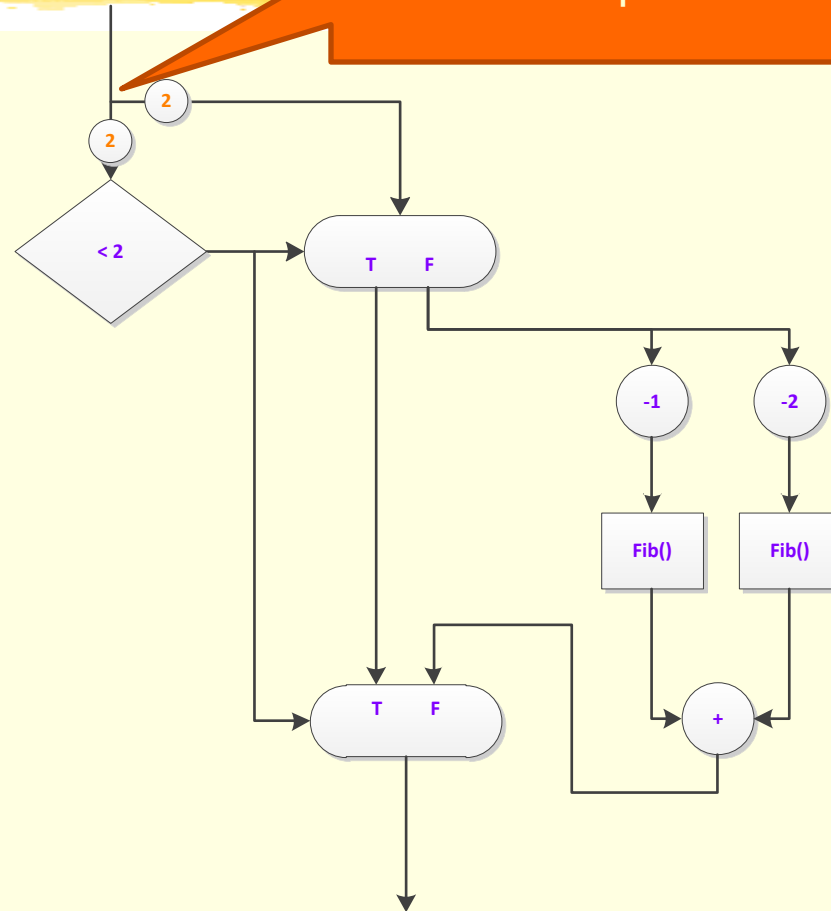
Let's begin: we have 1 token inserted for Fib(2)



An Example with Fibonacci

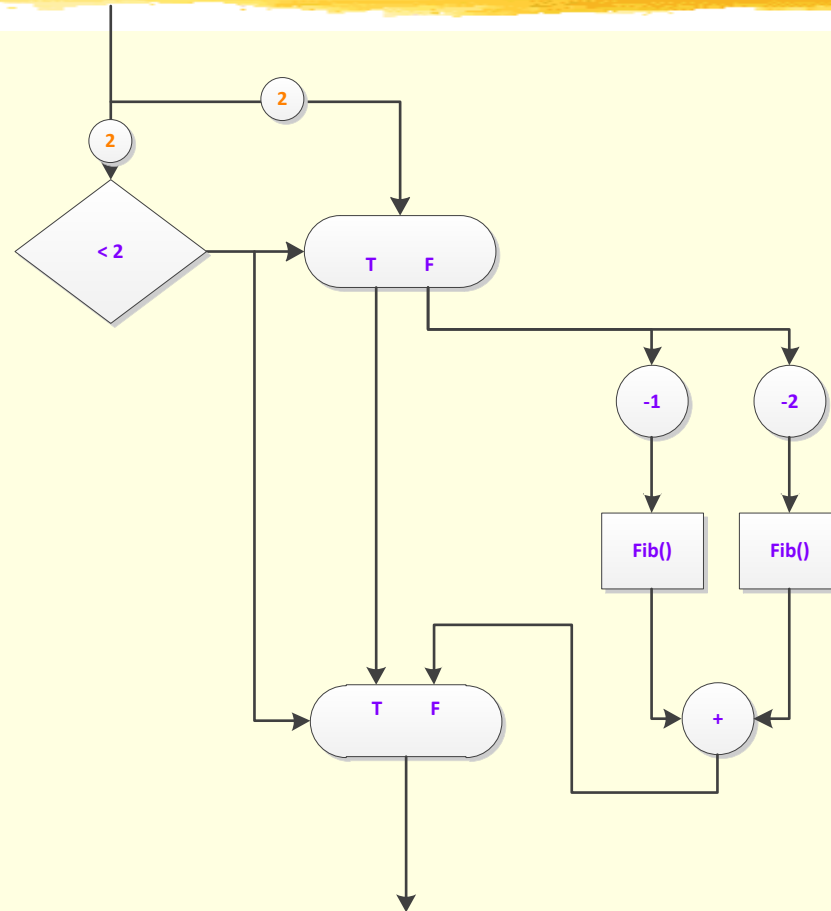
(Thanks to J.La)

It splits into two token here at a split node

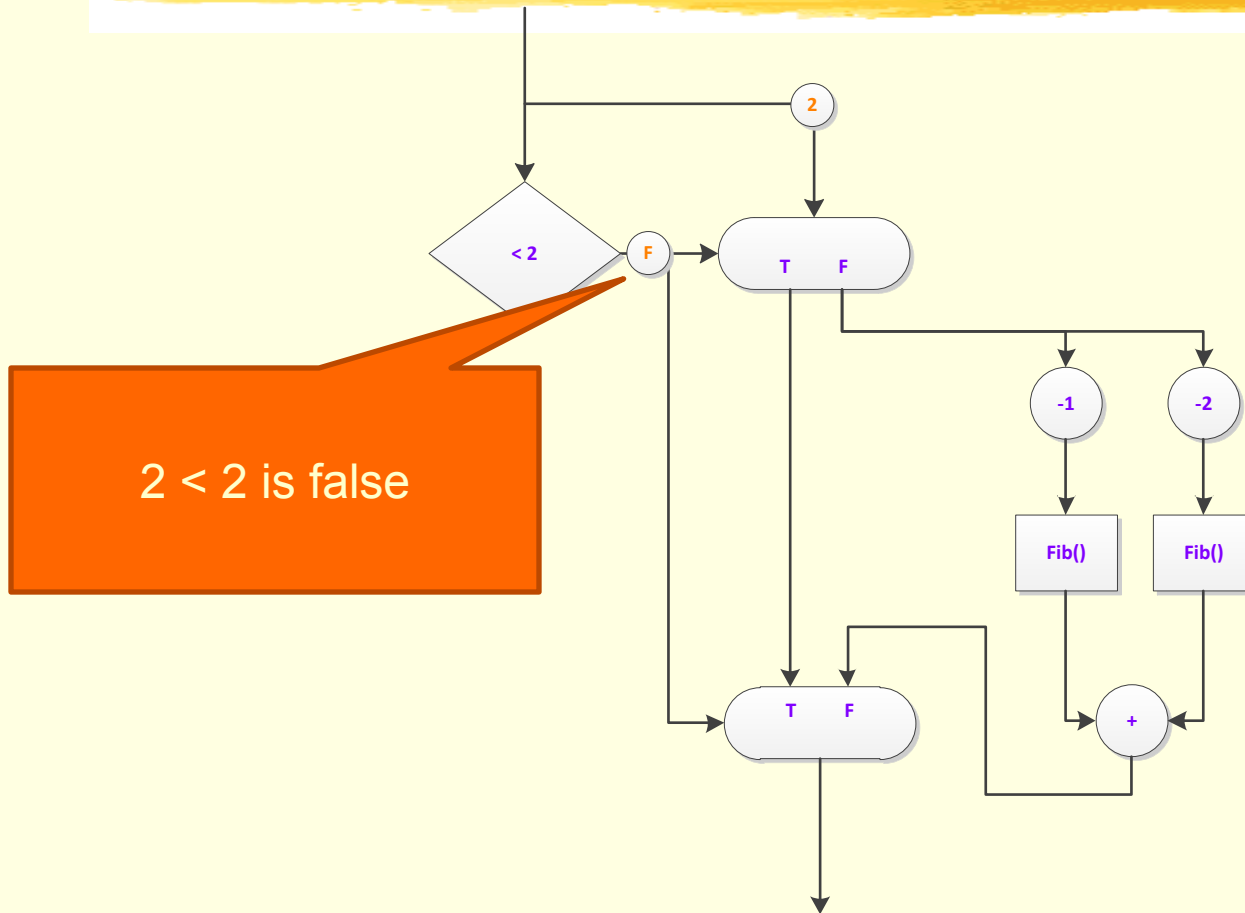


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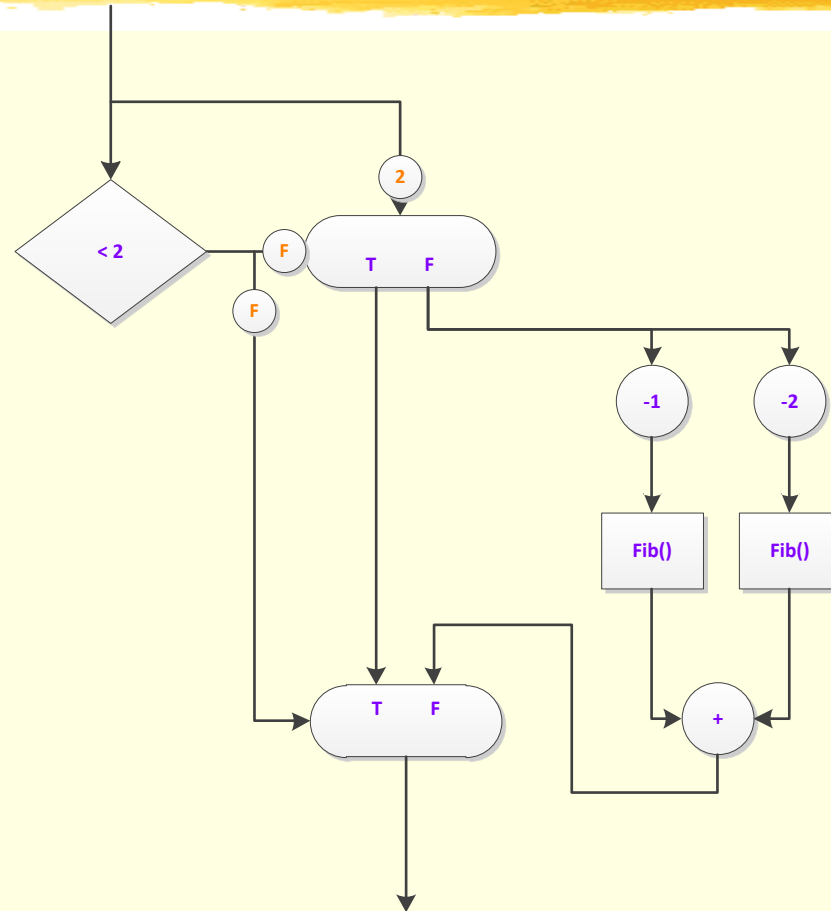


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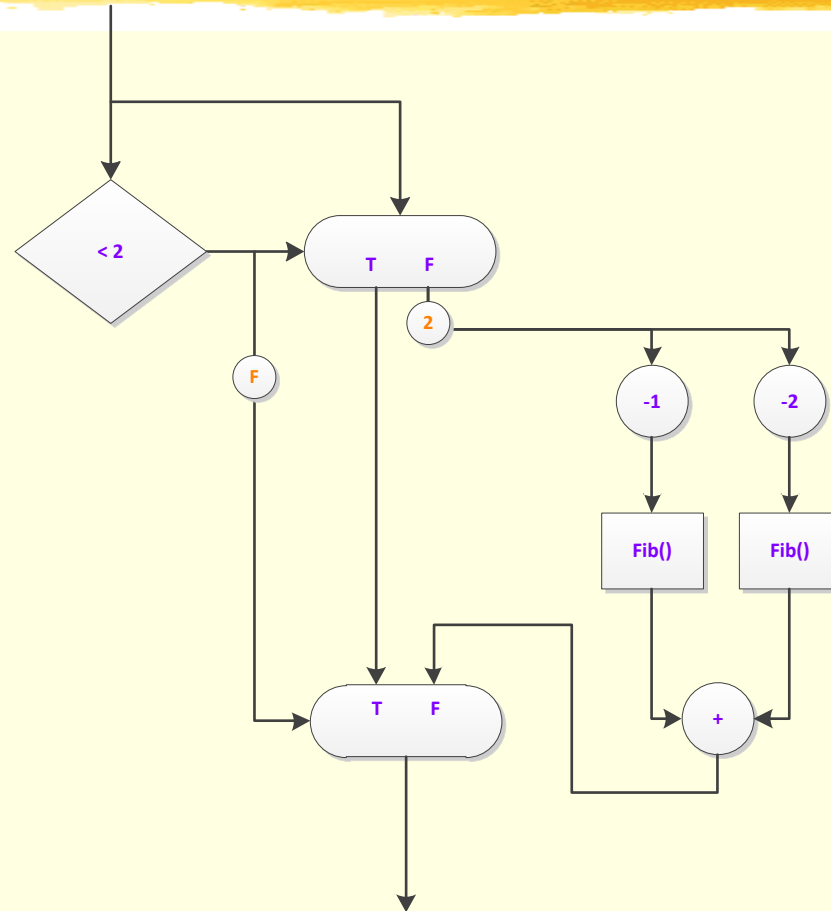
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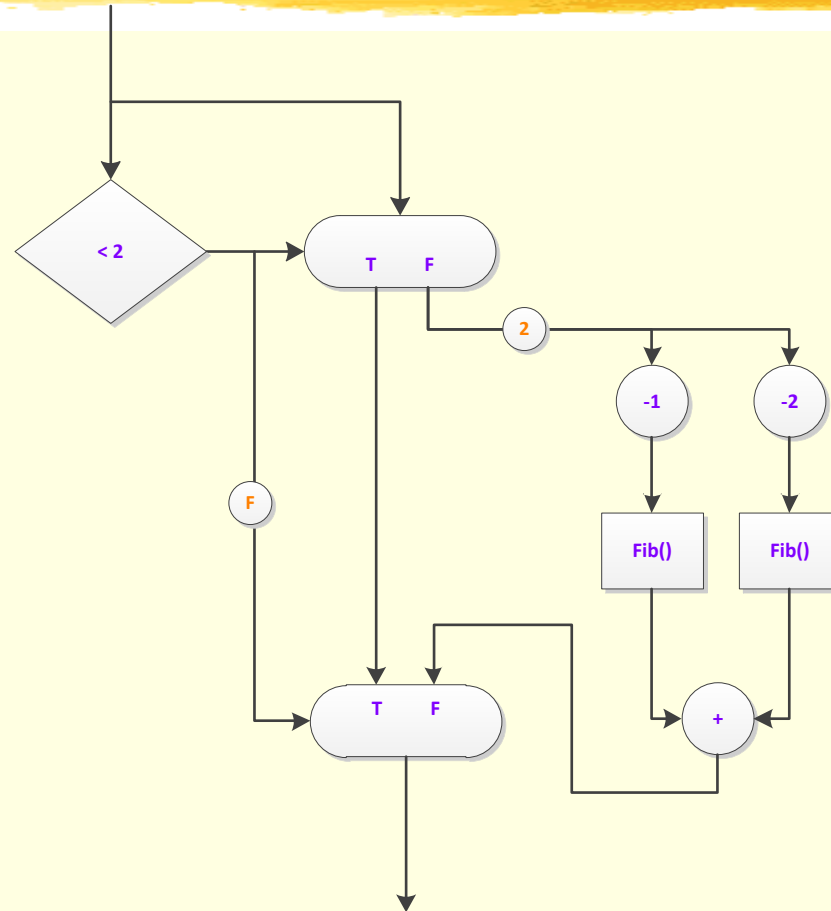
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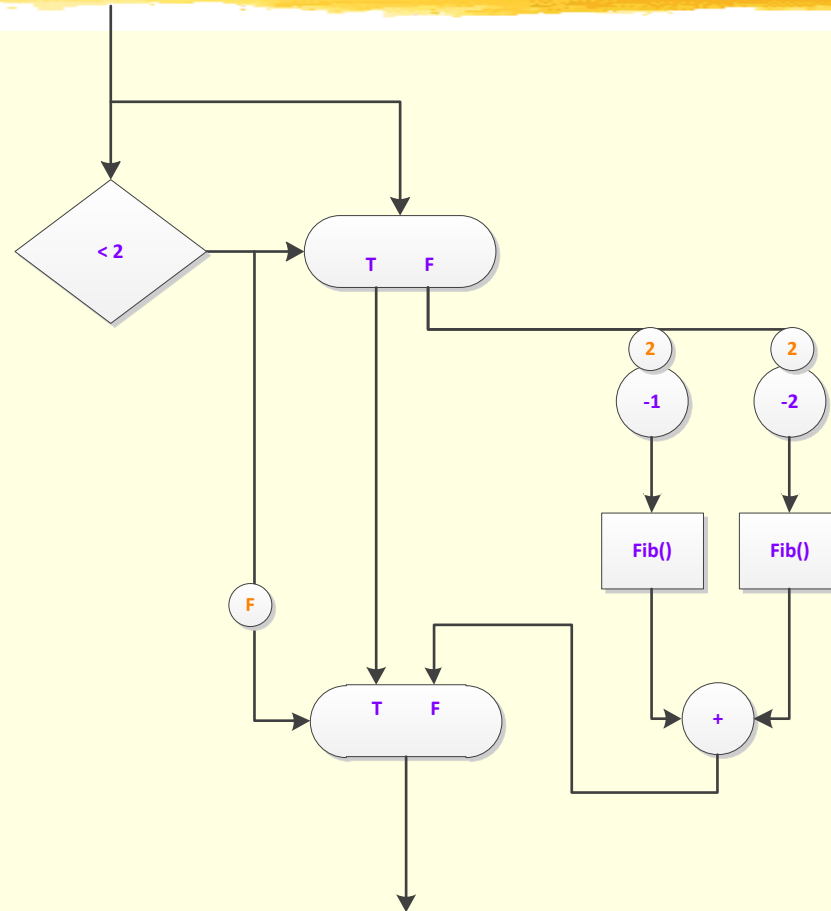
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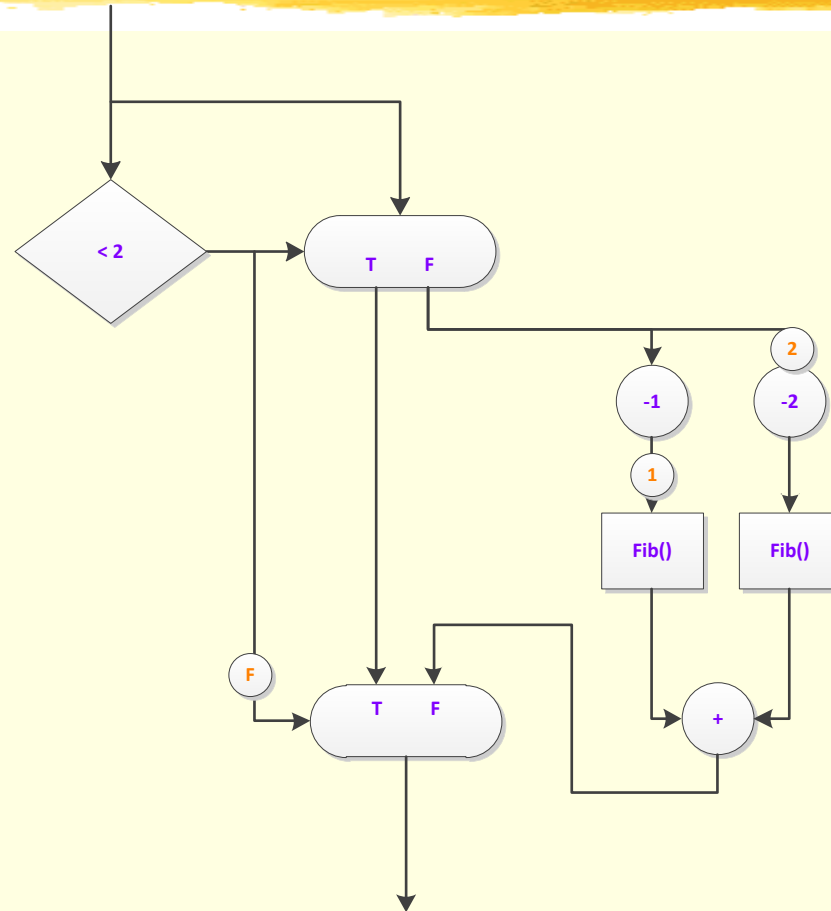


An Example with Fibonacci

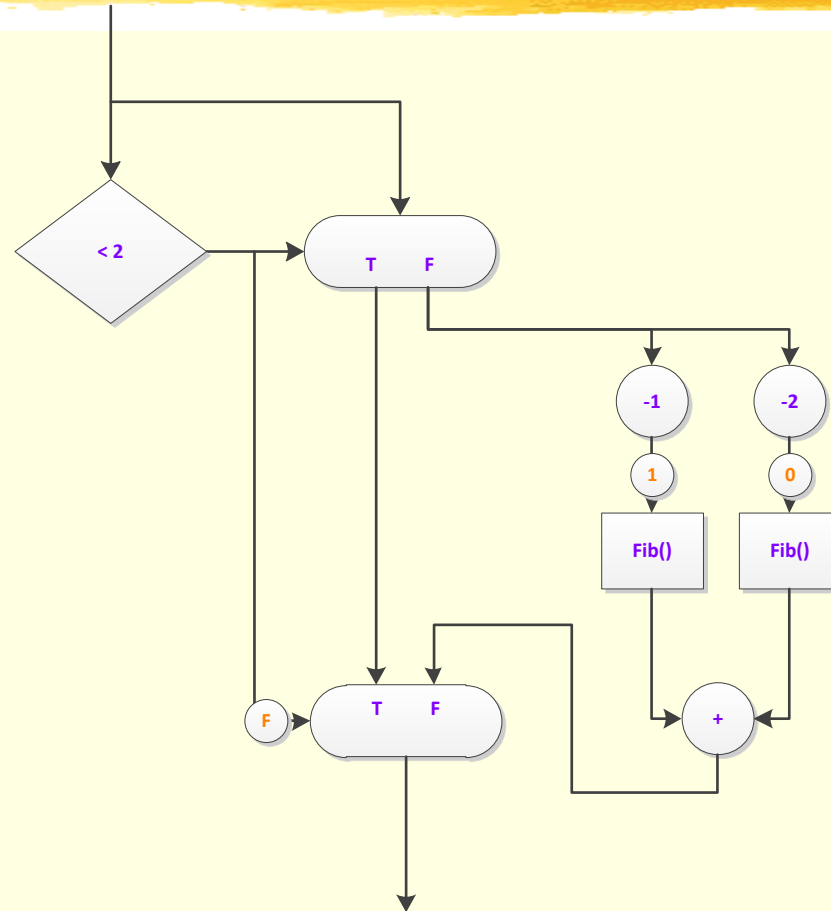
(Thanks to J.Landwehr)



An Example with Fibonacci (Thanks to J.Landwehr)



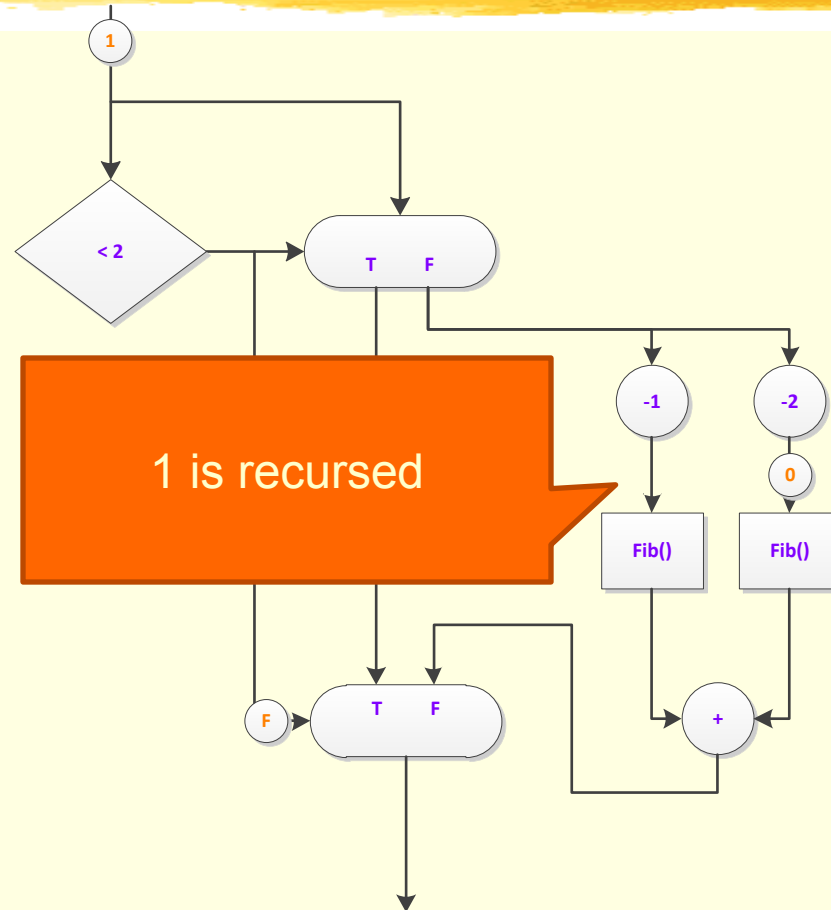
An Example with Fibonacci (Thanks to J.Landwehr)



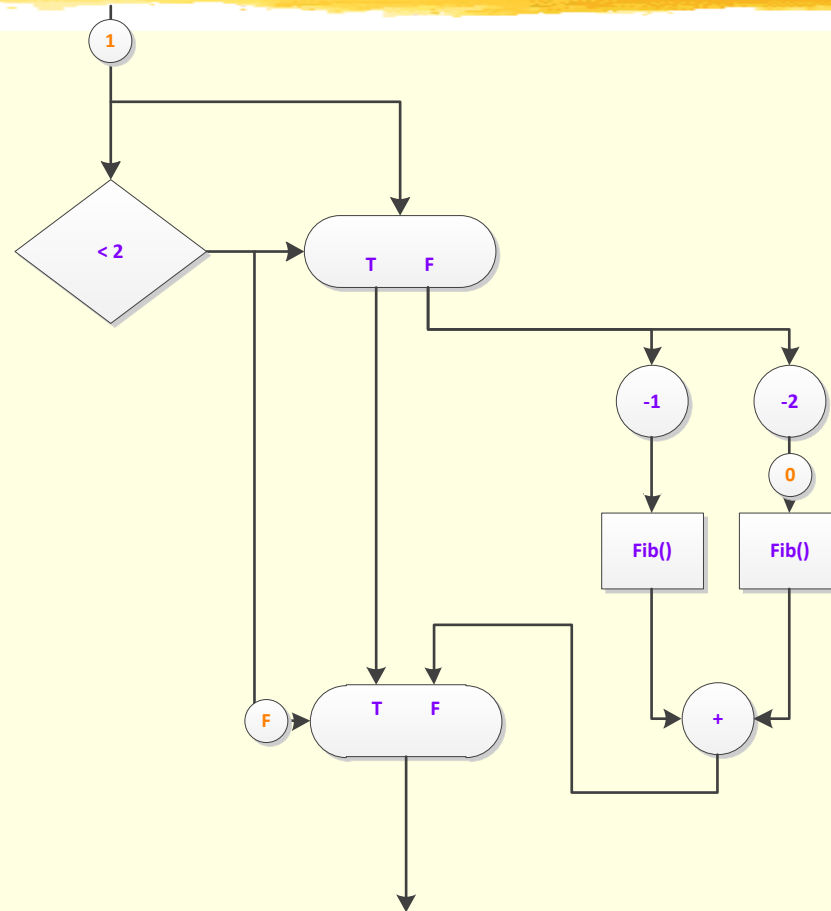
Recursively call the
Fib graph

An Example with Fibonacci

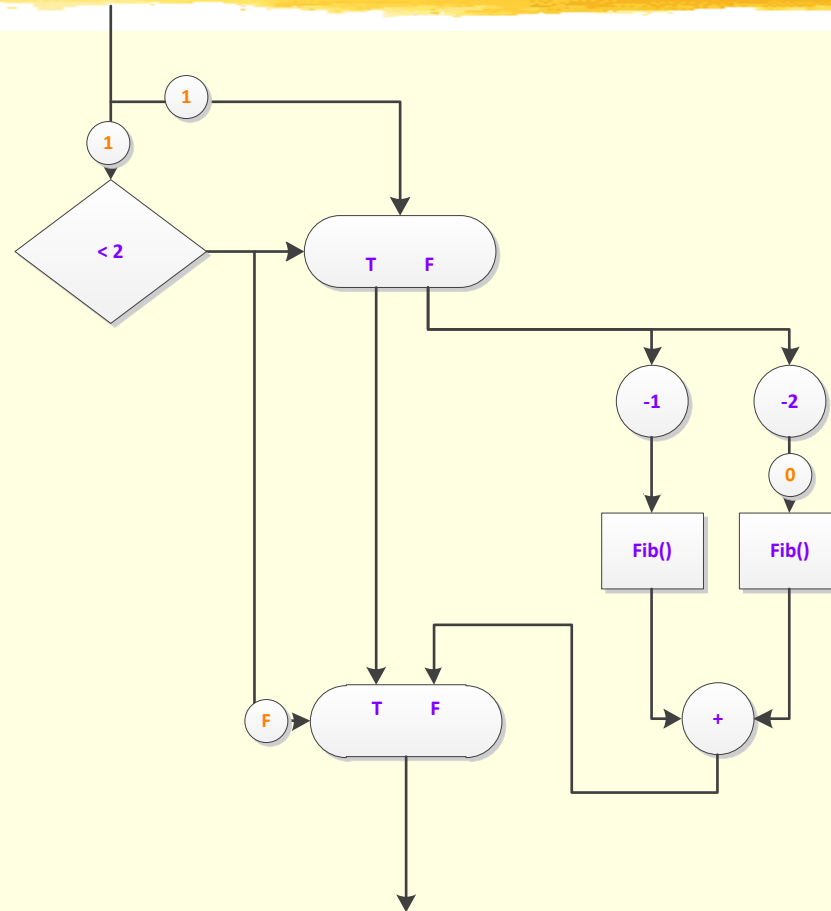
(Thanks to J.Landwehr)



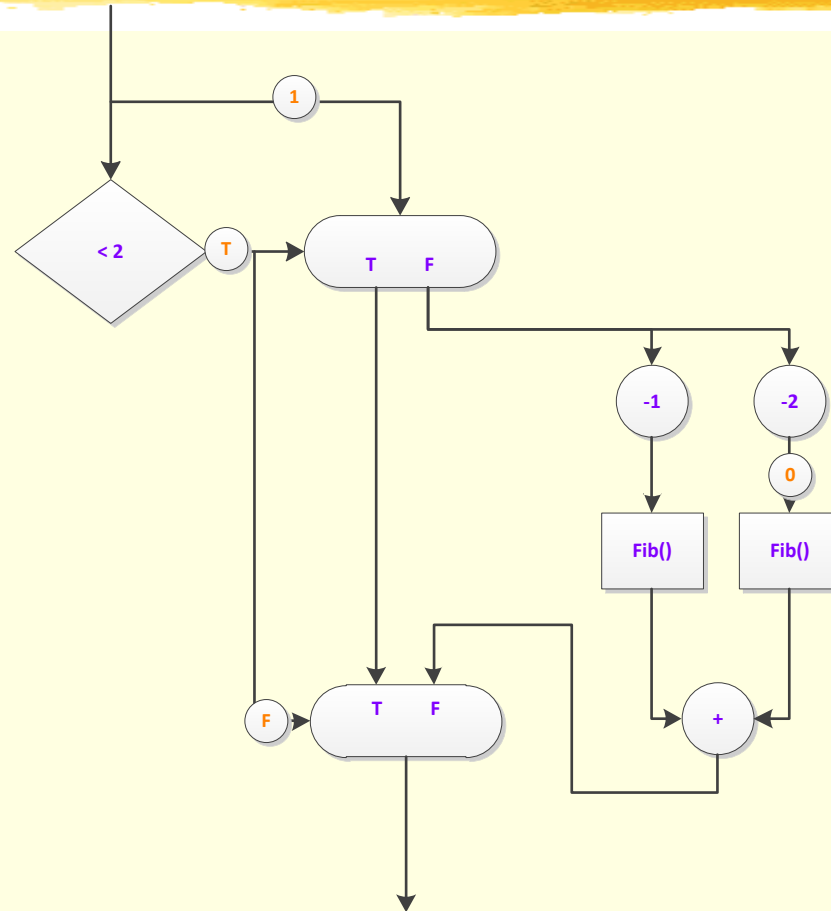
68



69

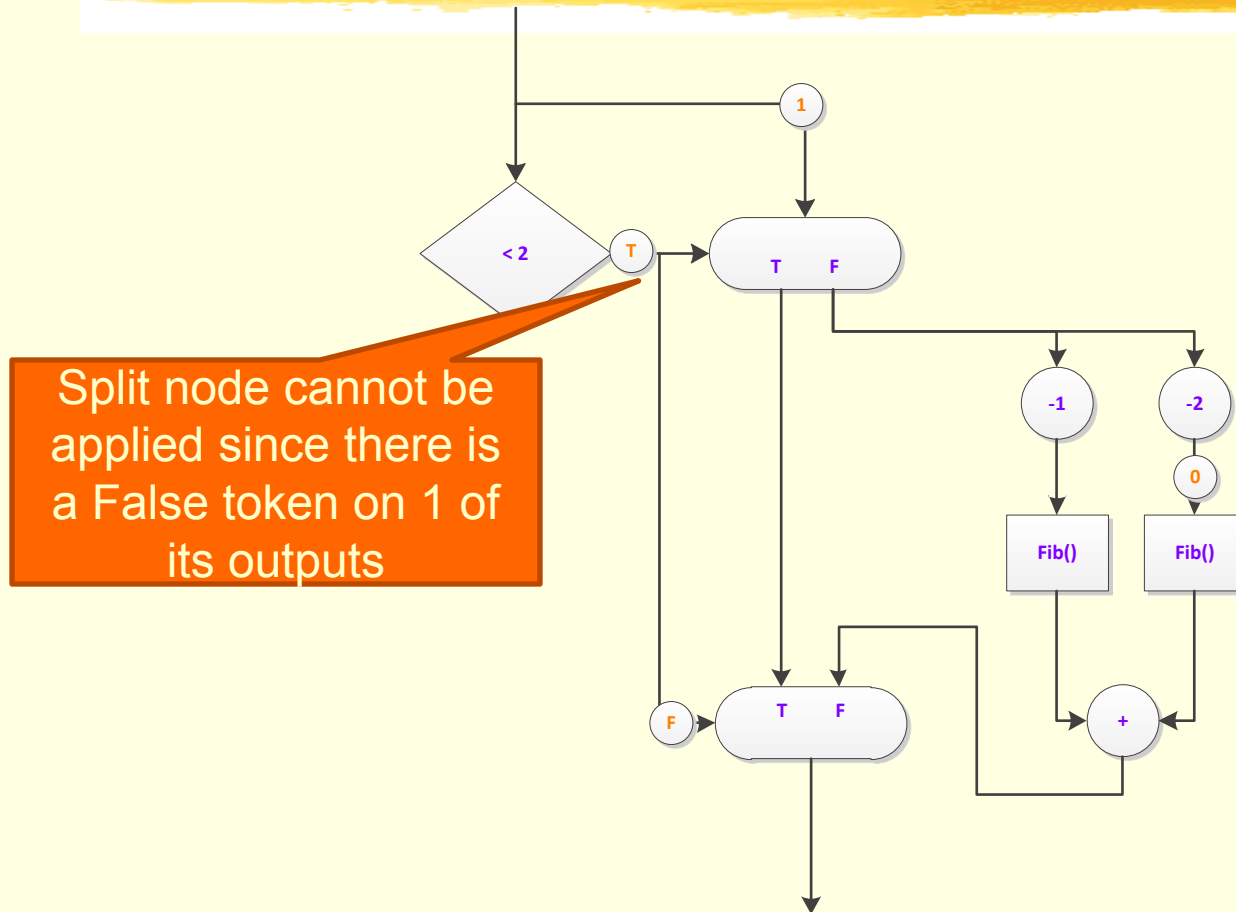


70

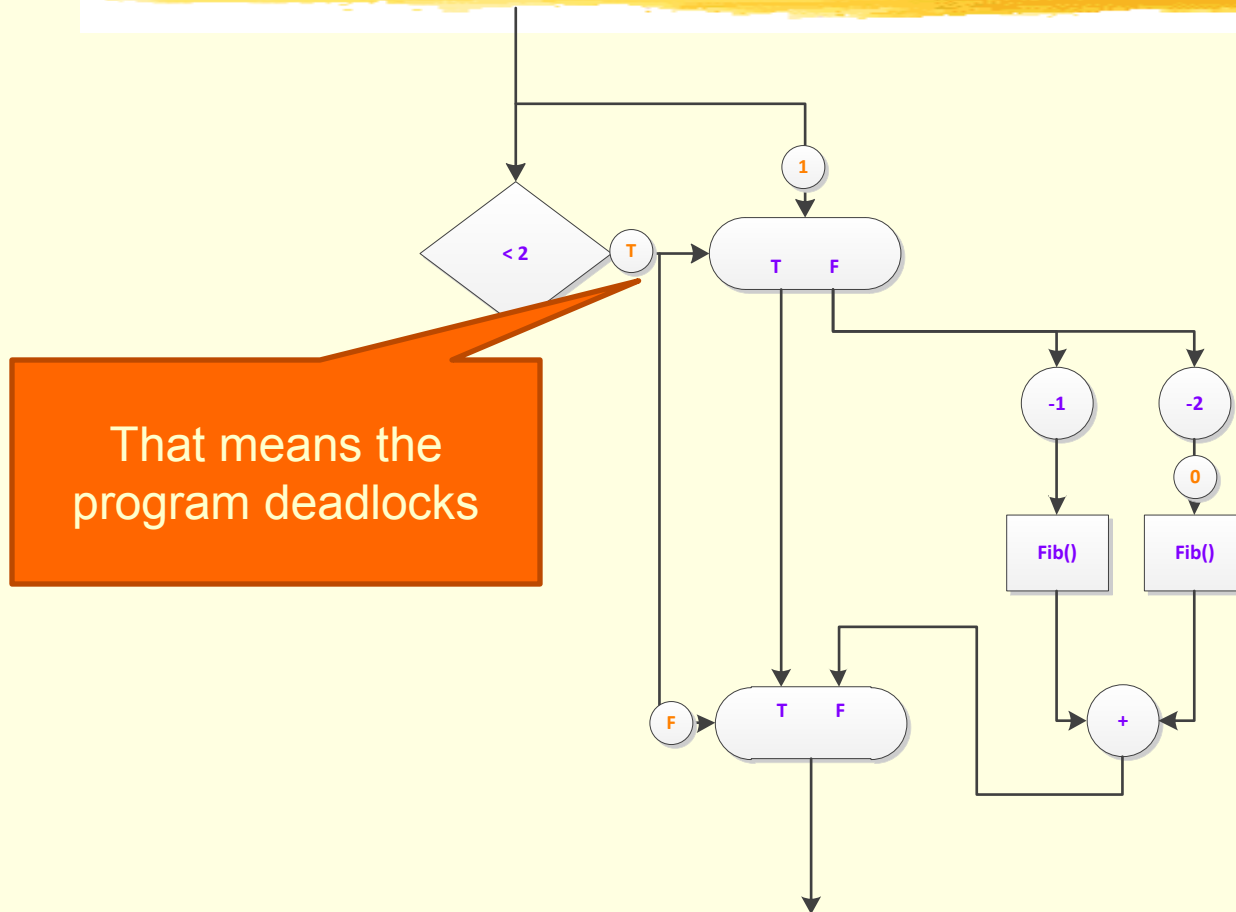


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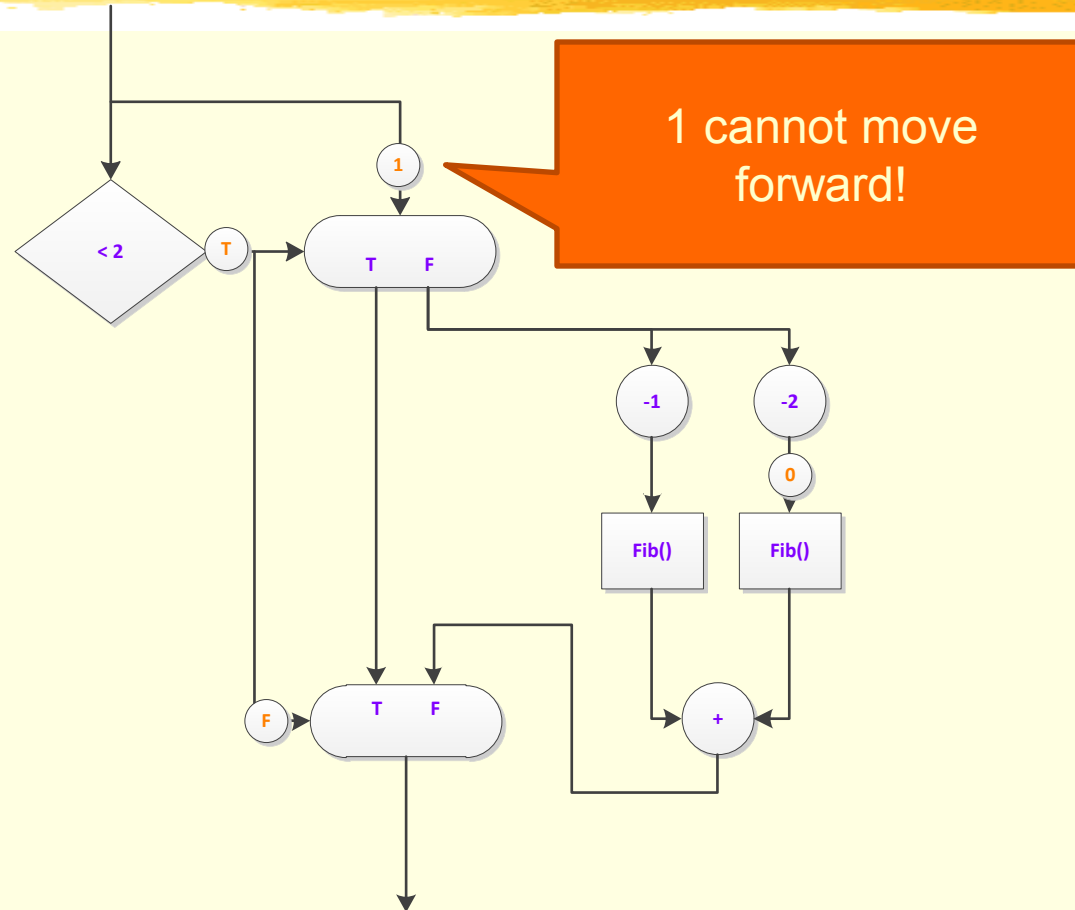


An Example with Fibonacci (Thanks to J.Landwehr)

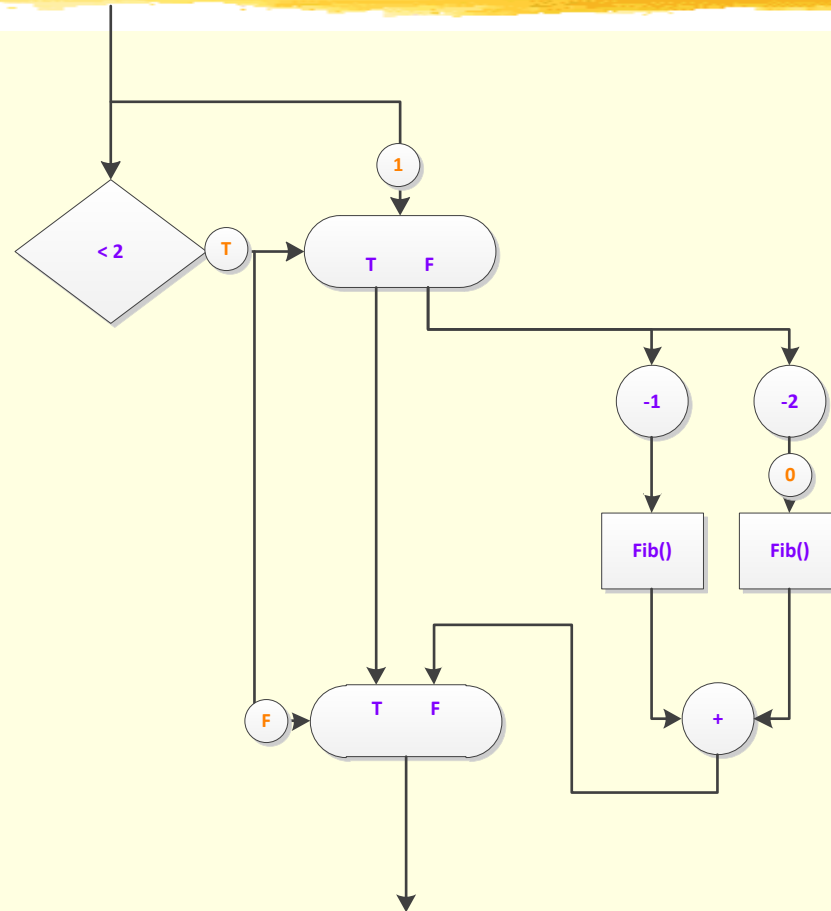


An Example with Fibonacci

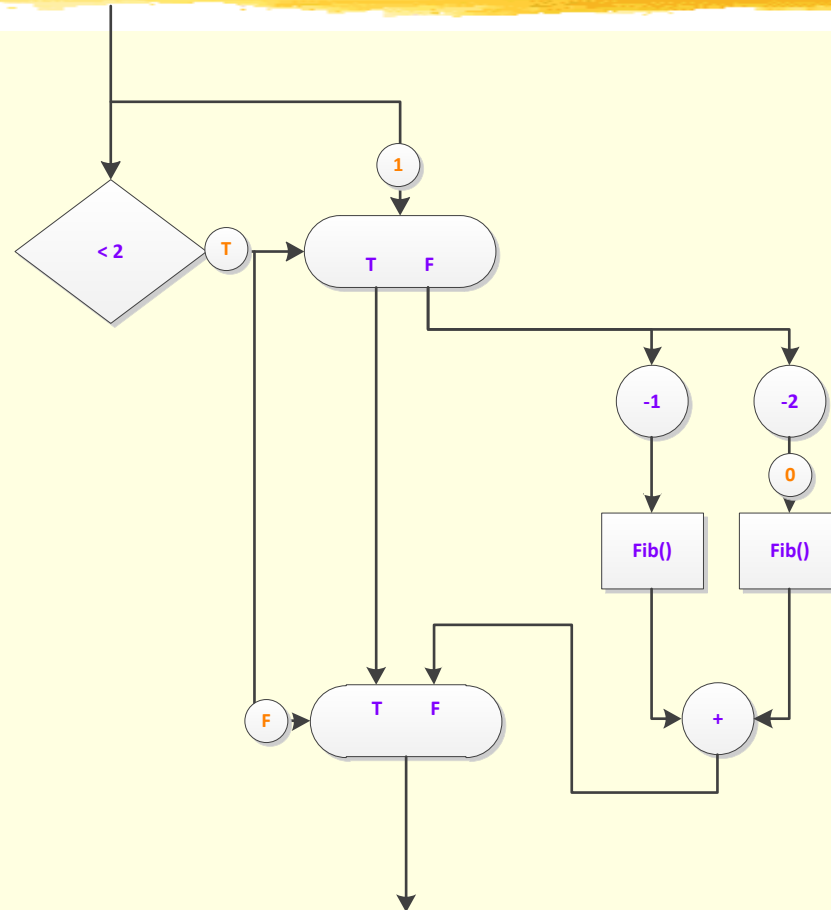
(Thanks to J.Landwehr)



An Example with Fibonacci (Thanks to J.Landwehr)



An Example with Fibonacci (Thanks to J.Landwehr)



Analysis: static dataflow cannot handle arbitrary recursion unless it is tail recursive

Factorial

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

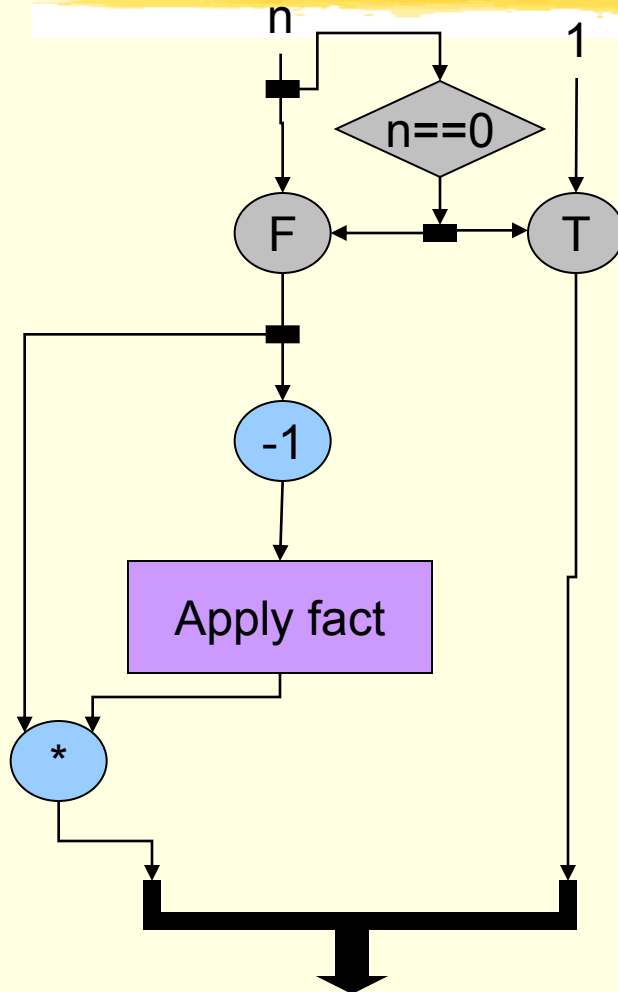
Normal Recursive

Tail Recursive

```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

Factorial

The Normal Version

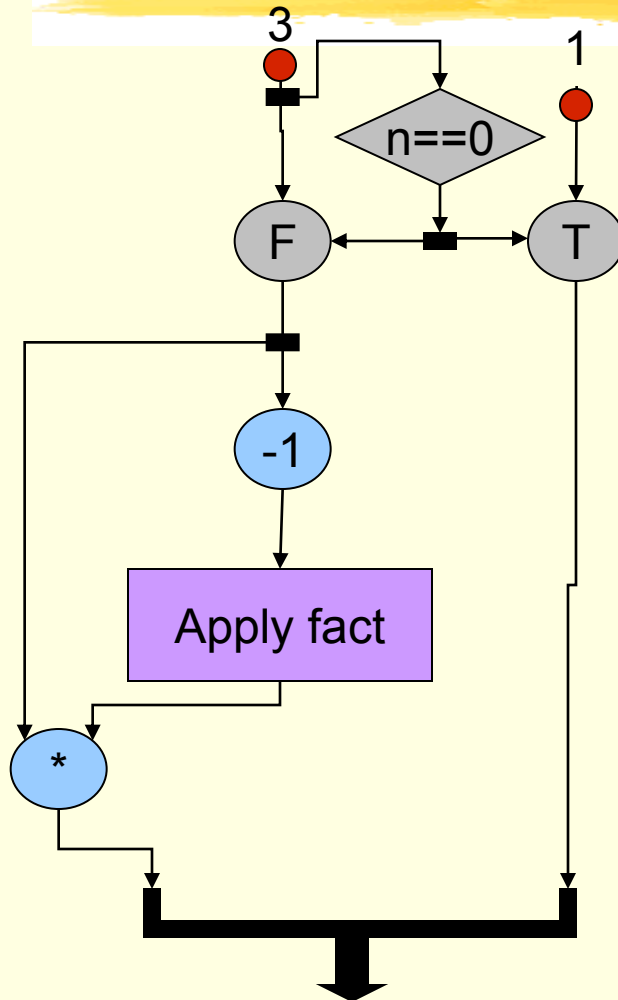


Hand Simulate fact(3)

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

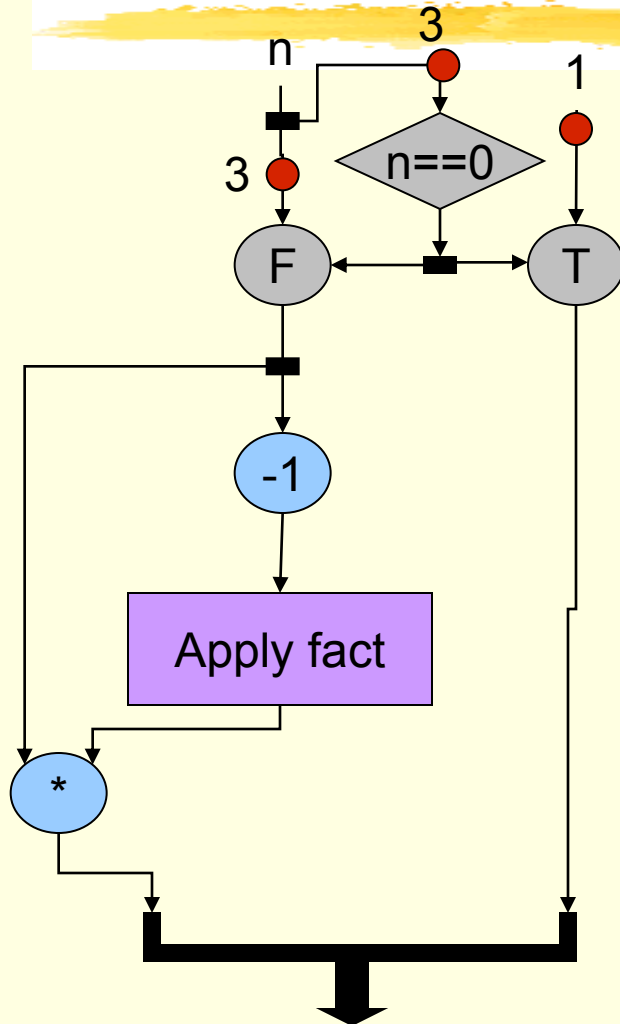


Hand Simulate fact(3)

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

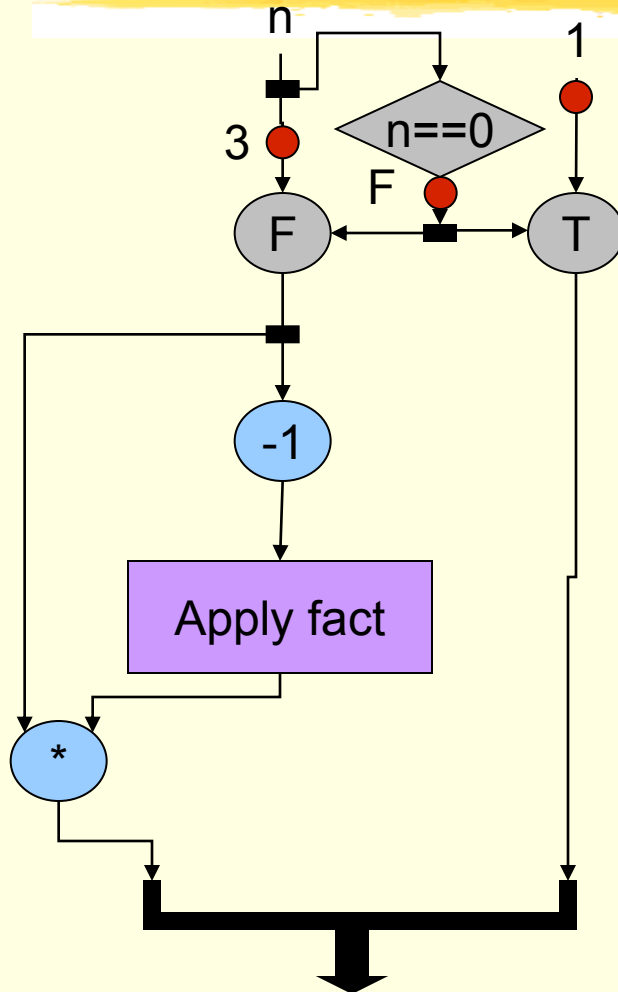


Hand Simulate fact(3)

```
long fact(n){
    if(n == 0) return 1;
    else return n * fact(n-1);
}
```

Factorial

The Normal Version

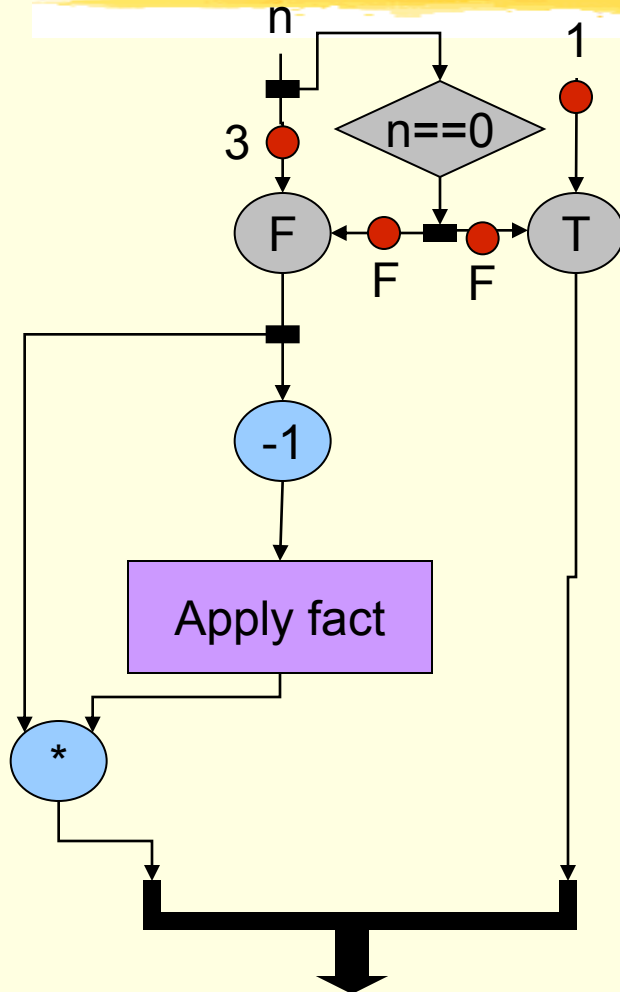


Hand Simulate fact(3)

```
long fact(n){
    if(n == 0) return 1;
    else return n * fact(n-1);
}
```

Factorial

The Normal Version

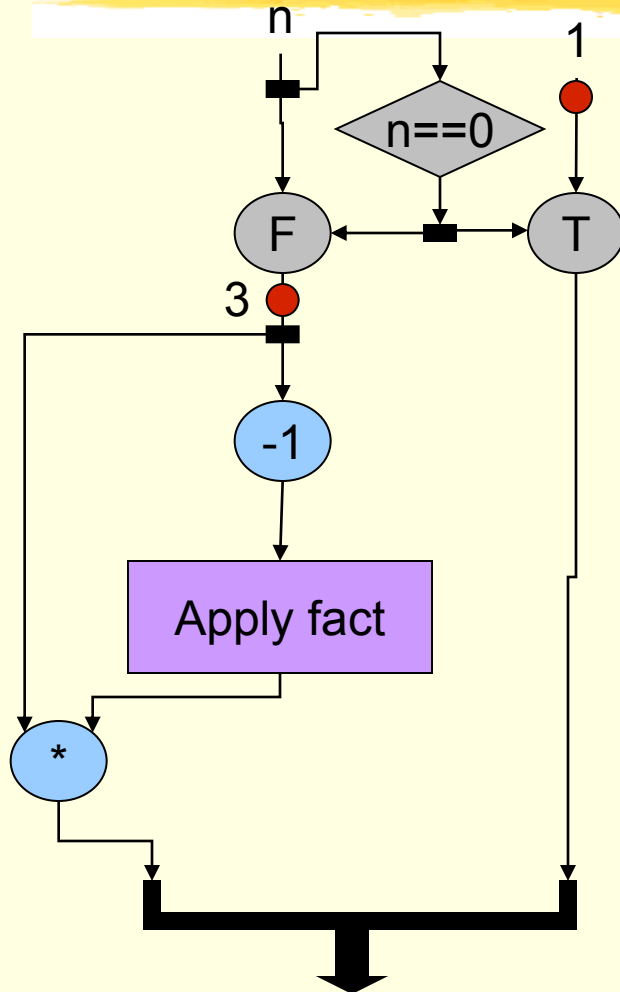


Hand Simulate fact(3)

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

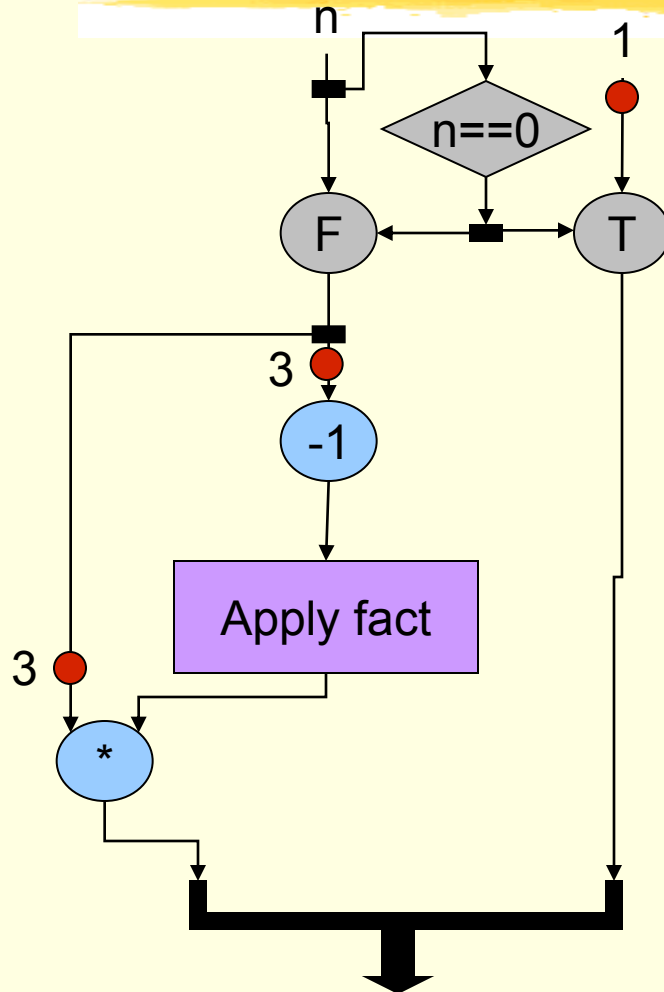


Hand Simulate fact(3)

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

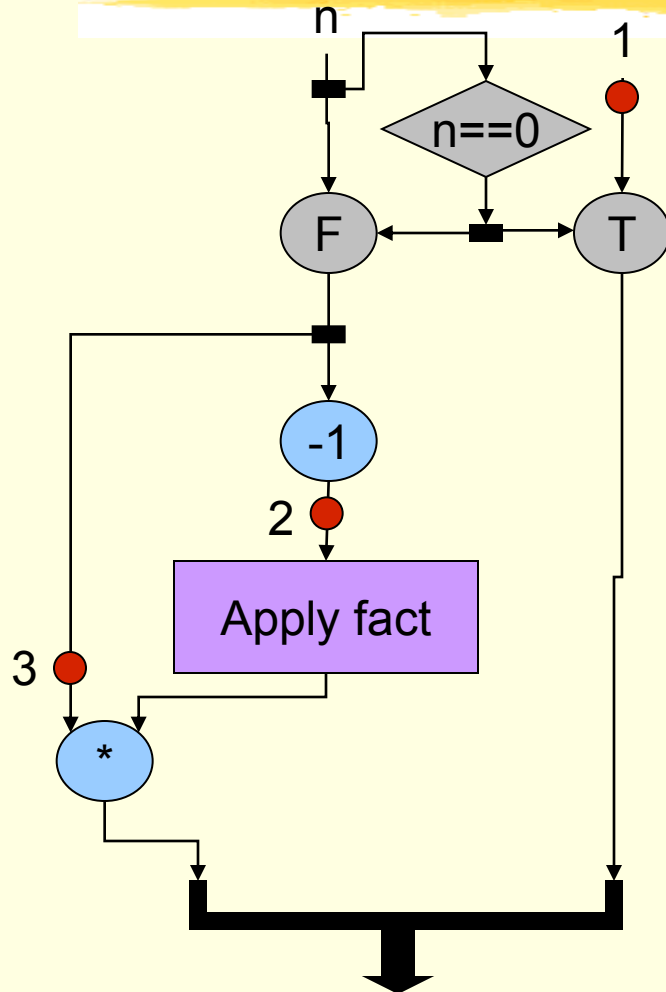


Hand Simulate fact(3)

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

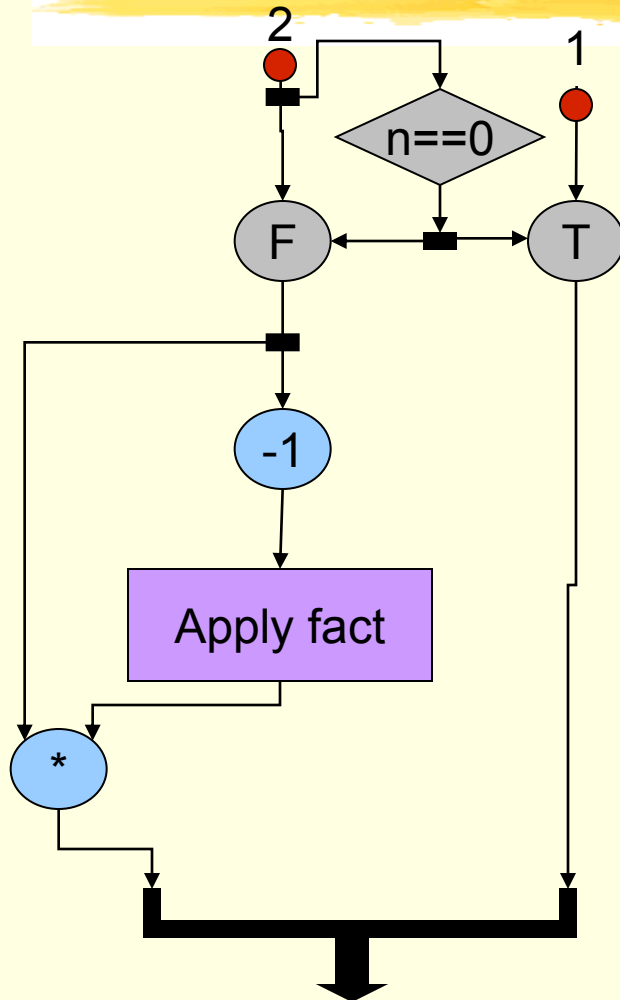


Hand Simulate fact(3)

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

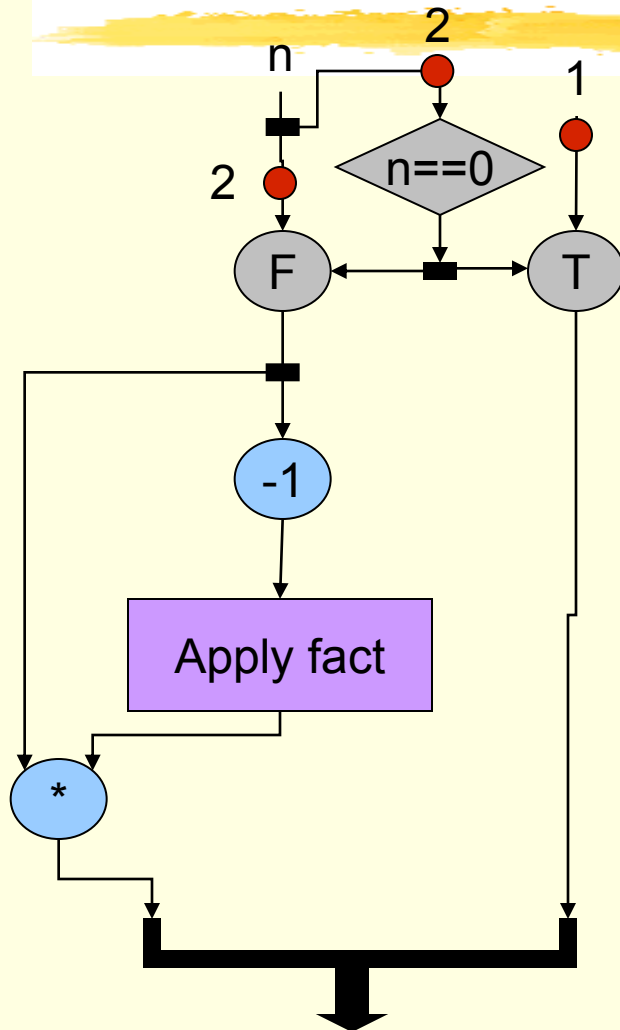


$3 * \text{fact}(2)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

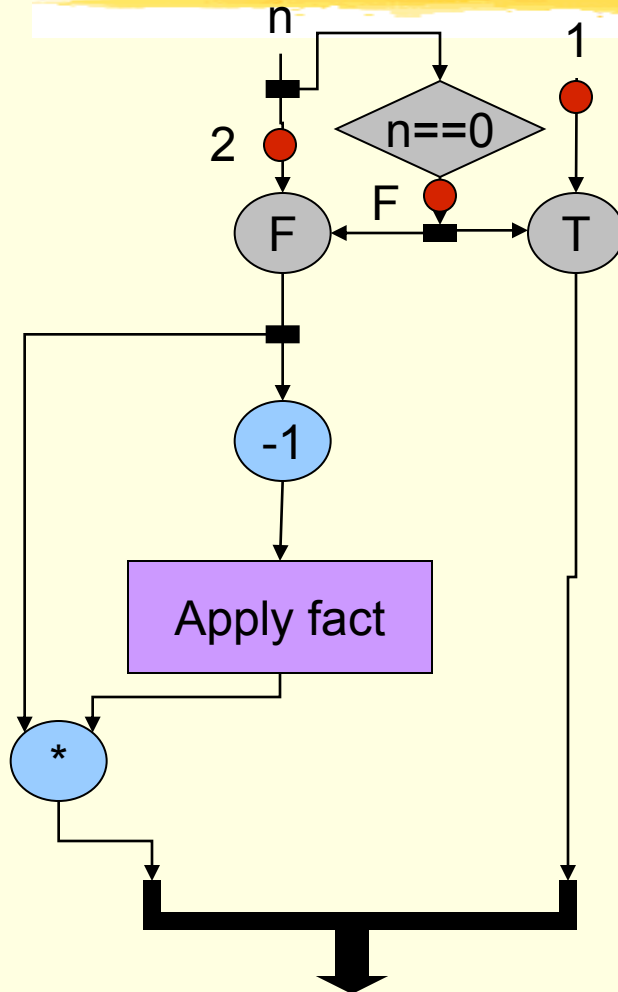


$3 * \text{fact}(2)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

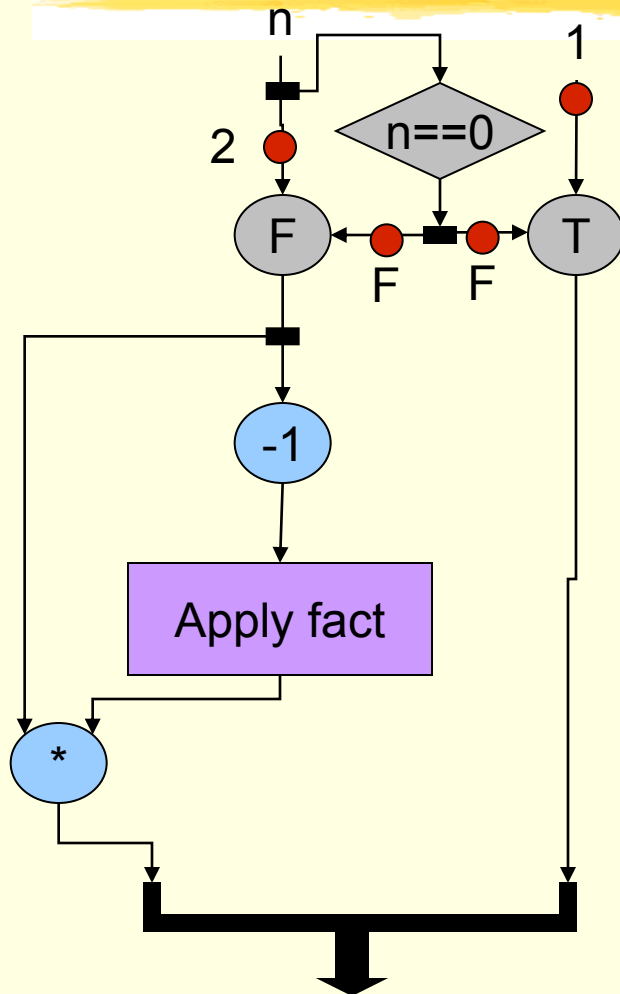


$3 * \text{fact}(2)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

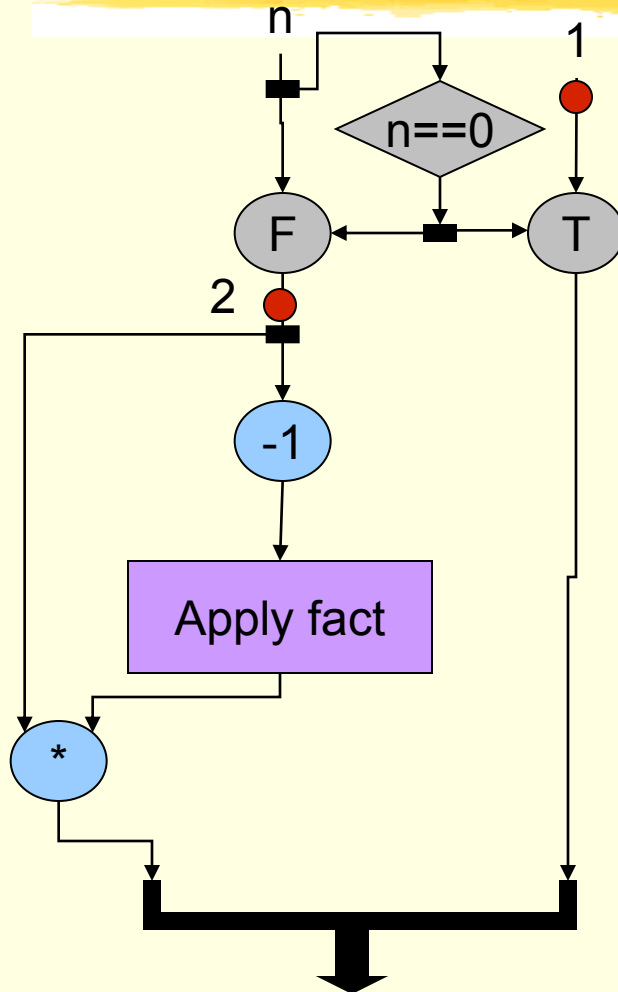


$3 * \text{fact}(2)$

```
long fact(n){
    if(n == 0) return 1;
    else return n * fact(n-1);
}
```

Factorial

The Normal Version

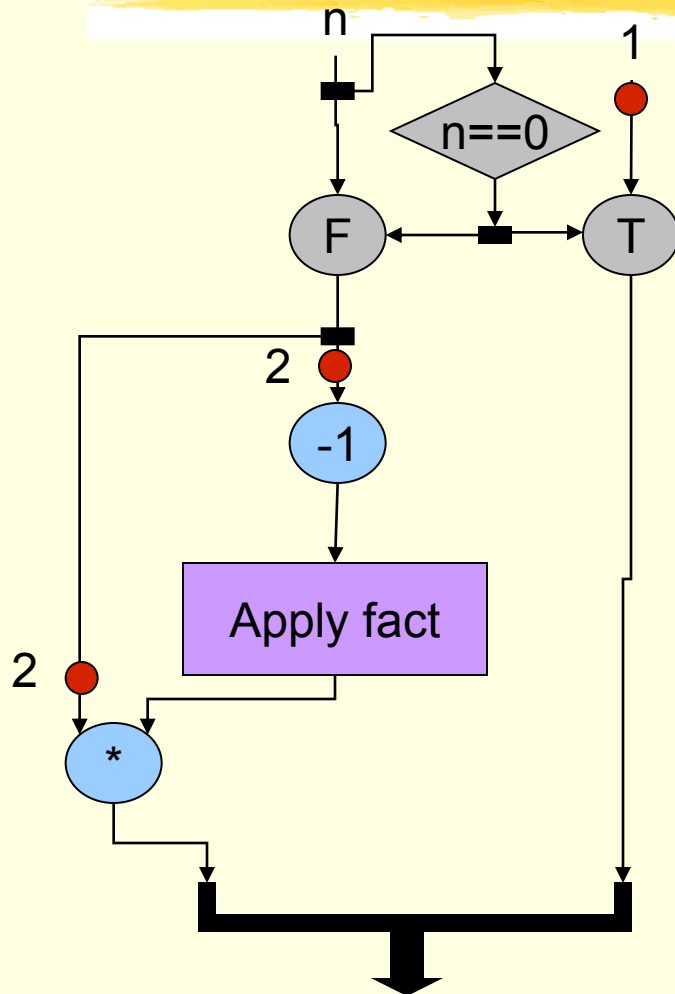


$3 * \text{fact}(2)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

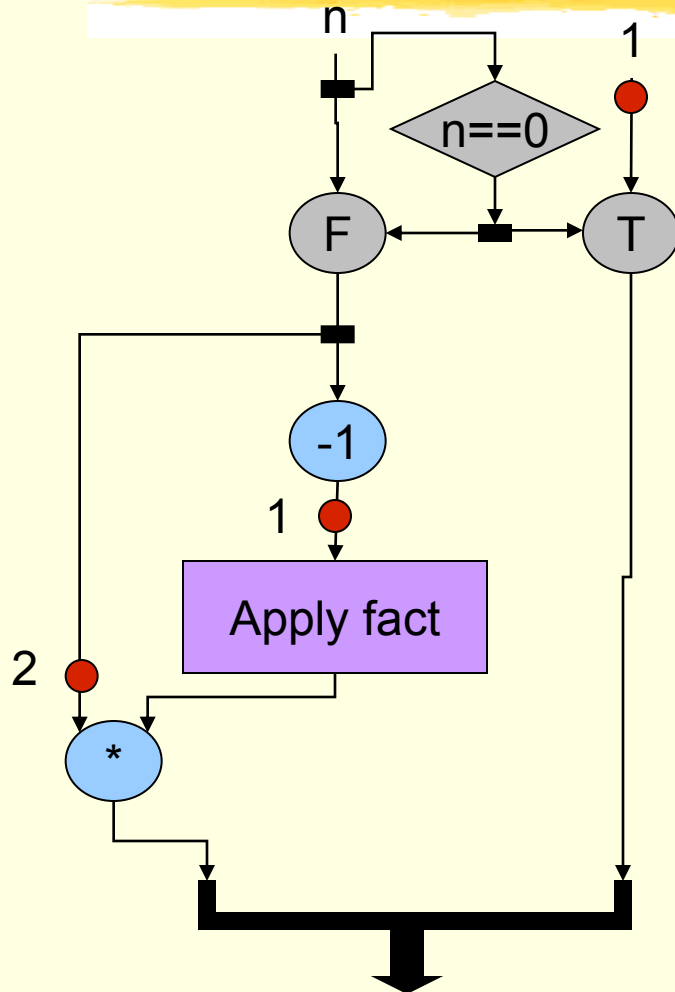


$3 * \text{fact}(2)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

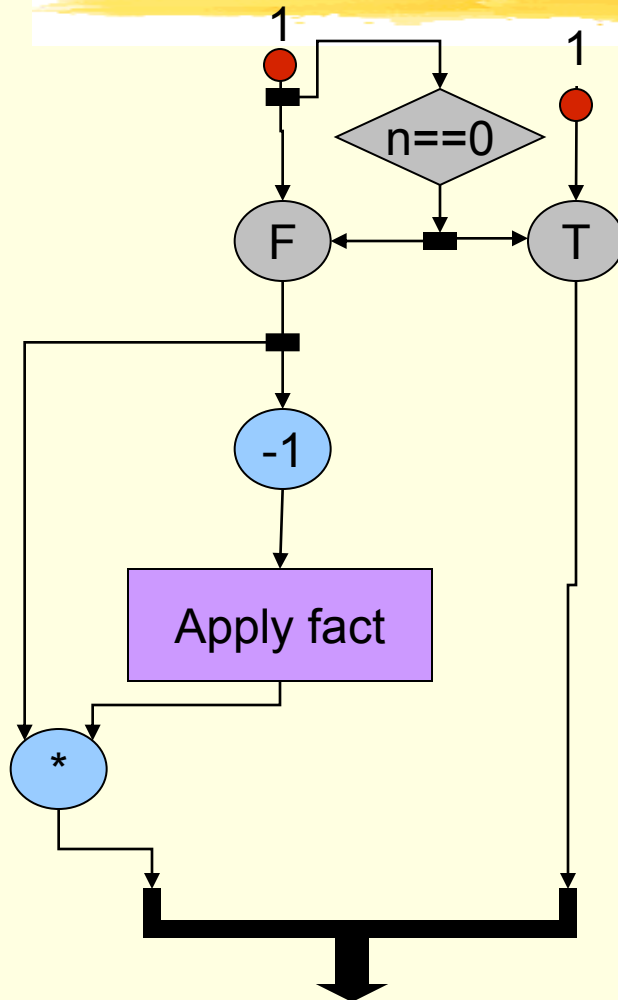


$3 * \text{fact}(2)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

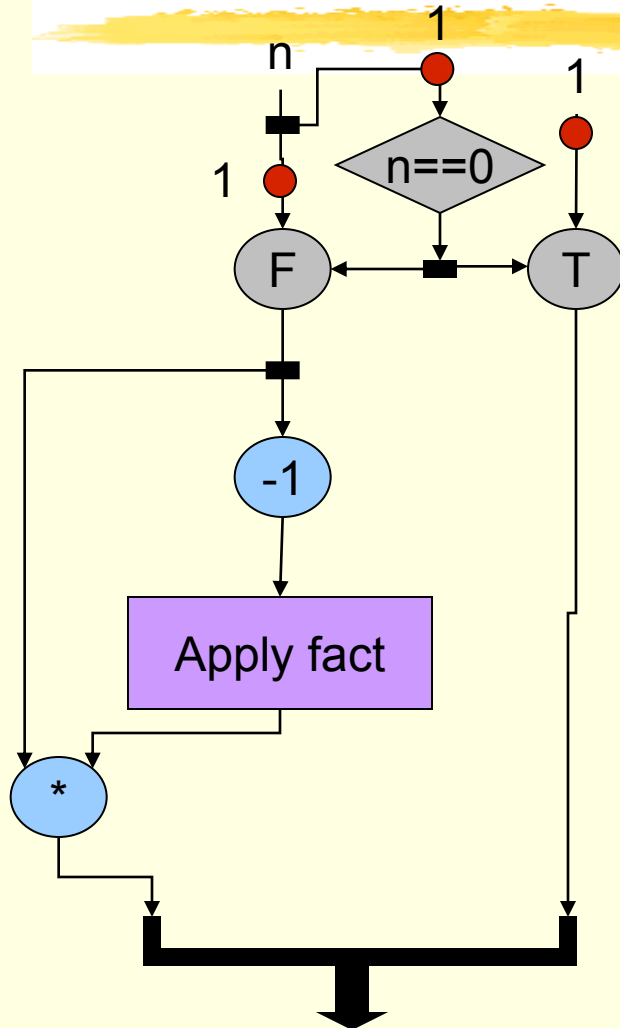


$3 * \text{fact}(2 * \text{fact}(1))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

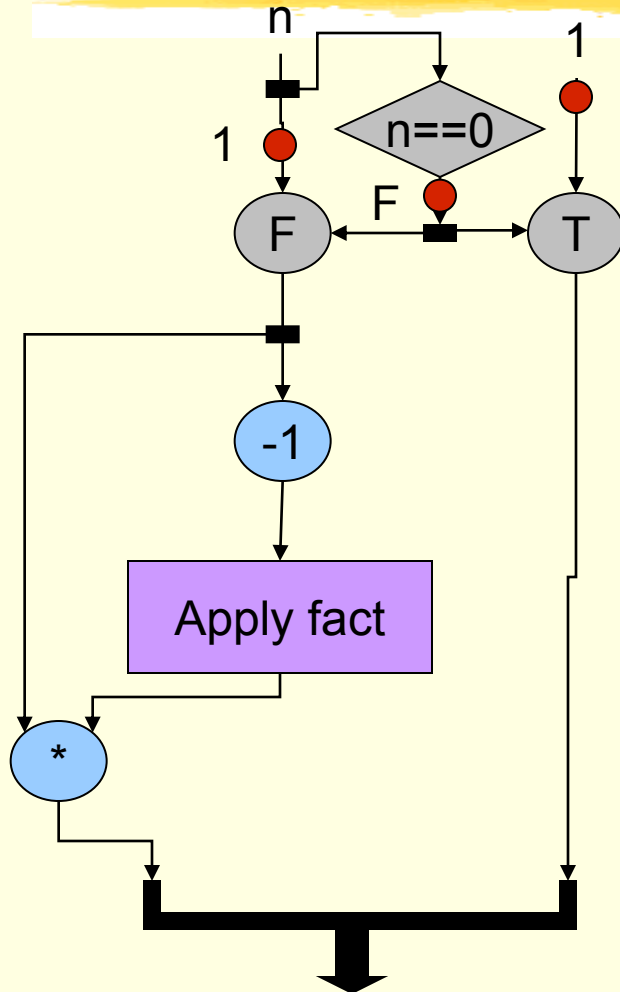


$3 * \text{fact}(2 * \text{fact}(1))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

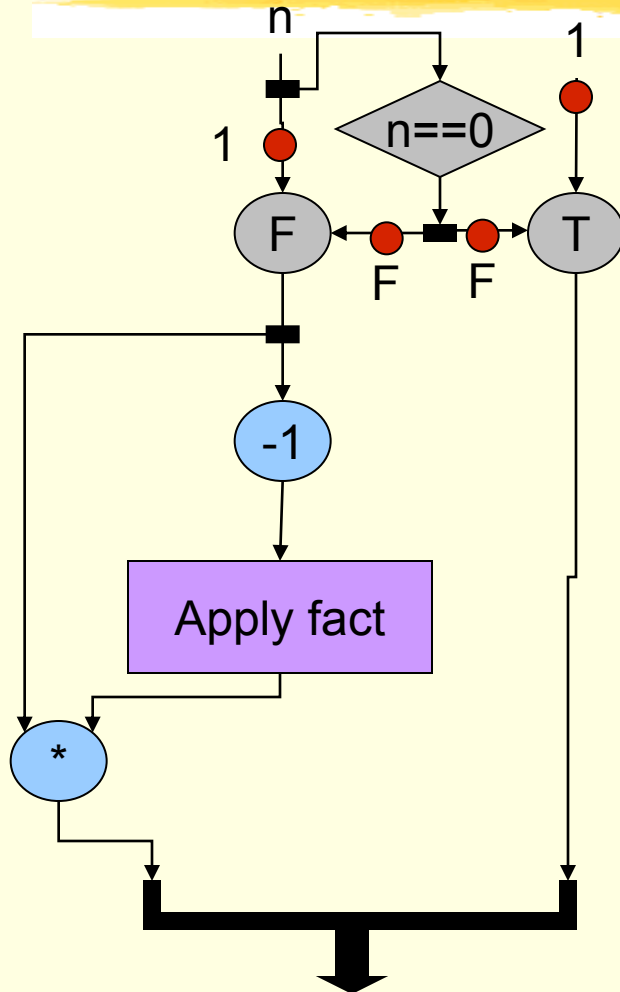


$3 * \text{fact}(2 * \text{fact}(1))$

```
long fact(n){
    if(n == 0) return 1;
    else return n * fact(n-1);
}
```

Factorial

The Normal Version

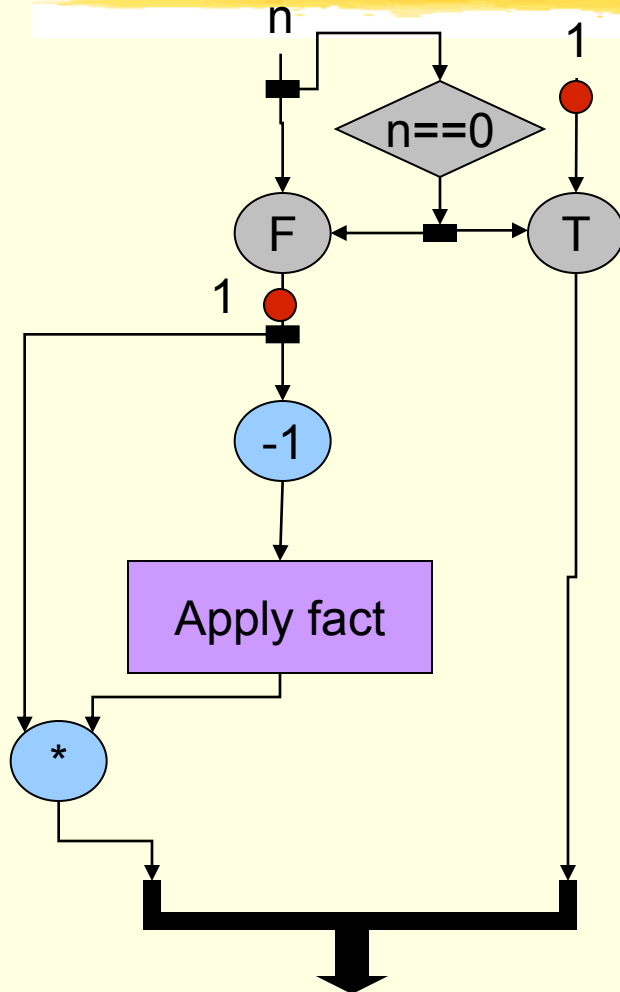


$3 * \text{fact}(2 * \text{fact}(1))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

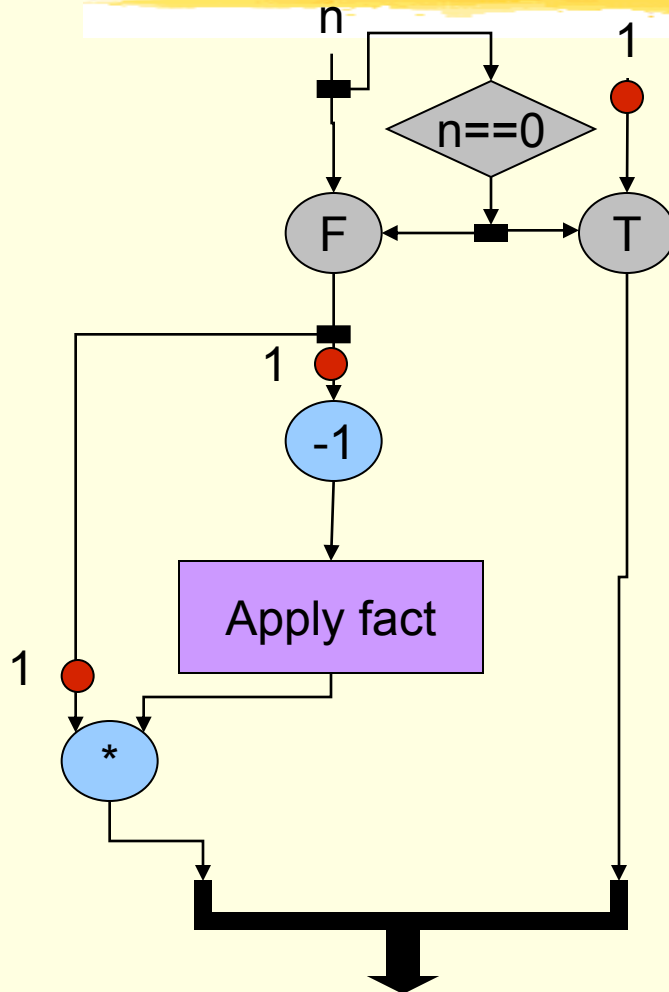


$3 * \text{fact}(2 * \text{fact}(1))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version



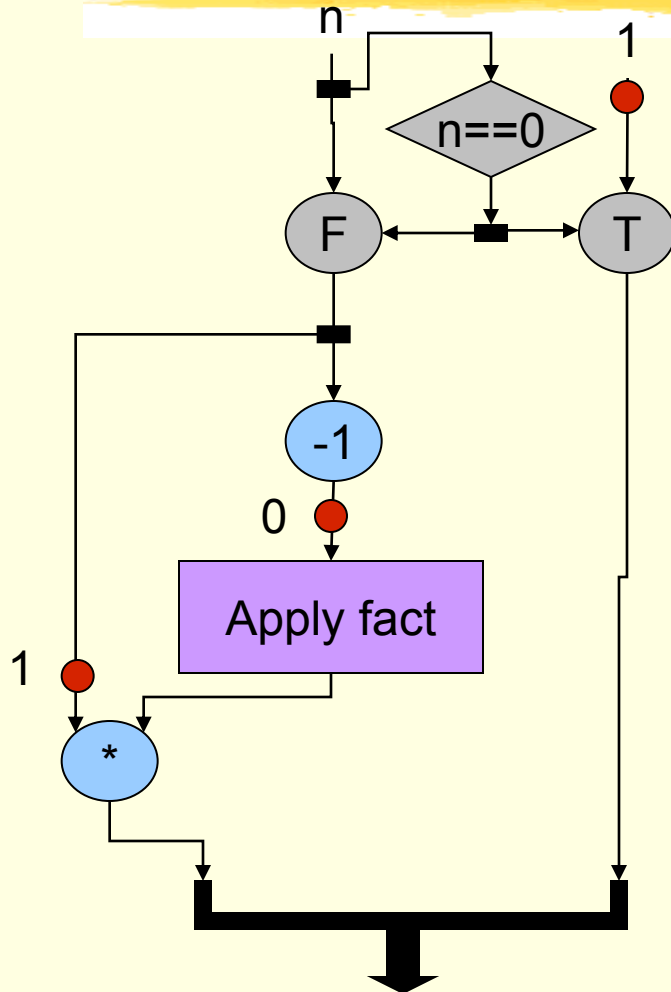
$3 * \text{fact}(2 * \text{fact}(1))$

```

long fact(n){
    if(n == 0) return 1;
    else return n * fact(n-1);
}
  
```

Factorial

The Normal Version

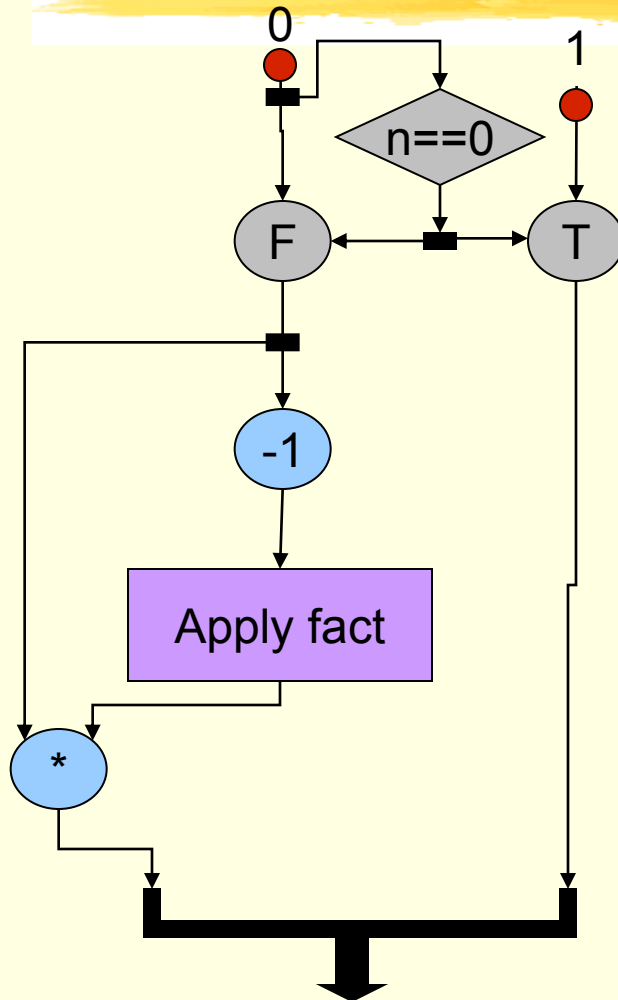


$3 * \text{fact}(2 * \text{fact}(1))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

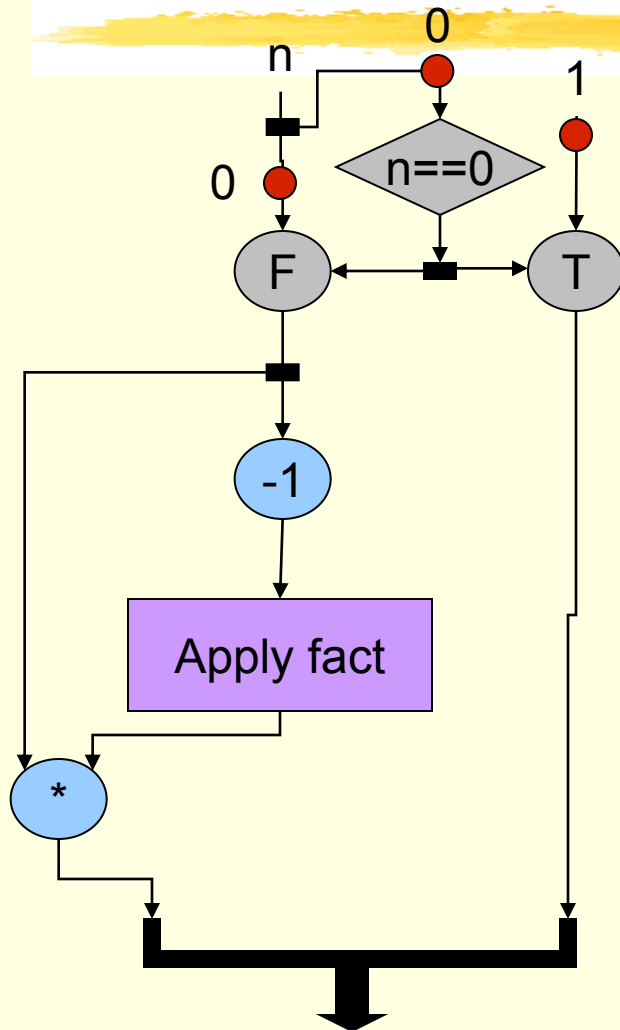


$3 * \text{fact}(2 * \text{fact}(1 * \text{fact}(0)))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

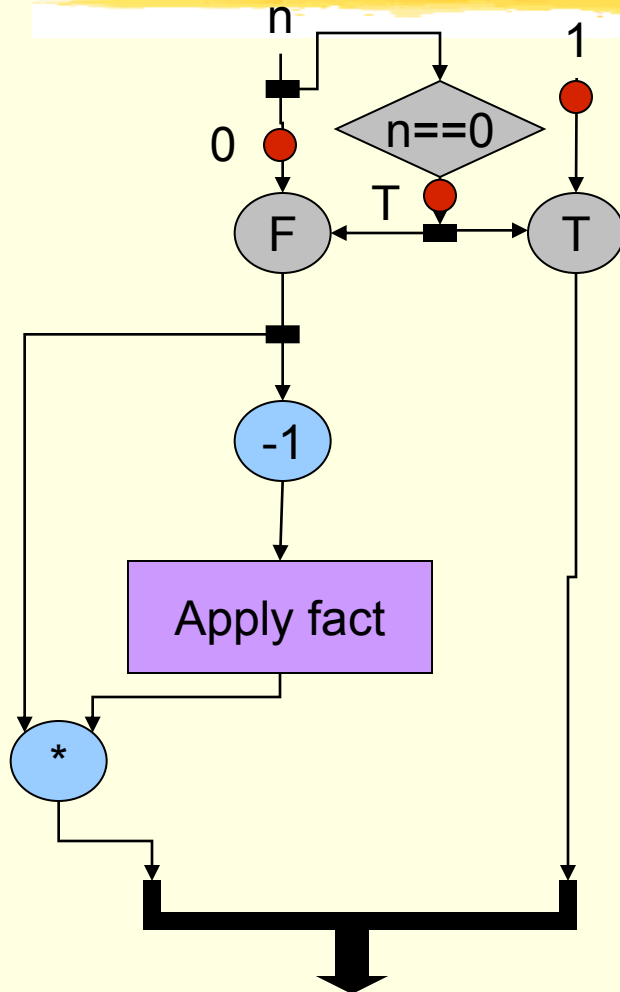


$3 * \text{fact}(2 * \text{fact}(1 * \text{fact}(0)))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

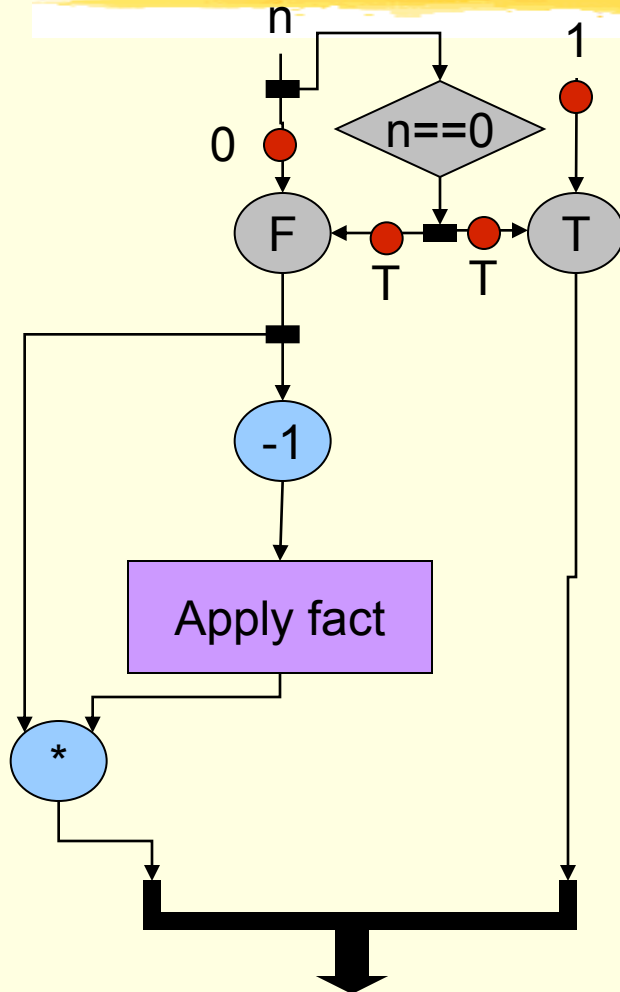


$3 * \text{fact}(2 * \text{fact}(1 * \text{fact}(0)))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

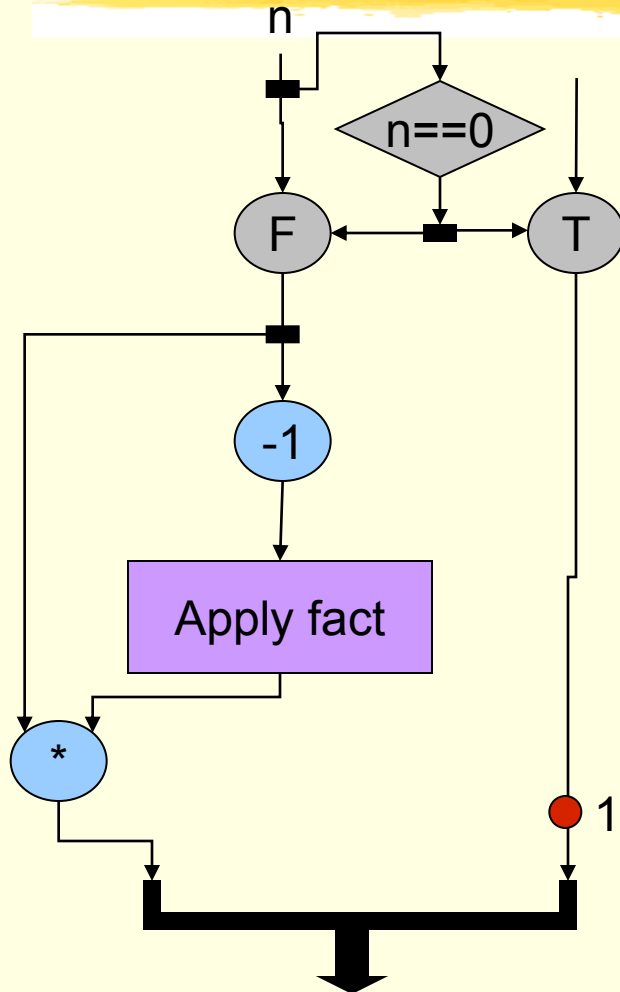


$3 * \text{fact}(2 * \text{fact}(1 * \text{fact}(0)))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

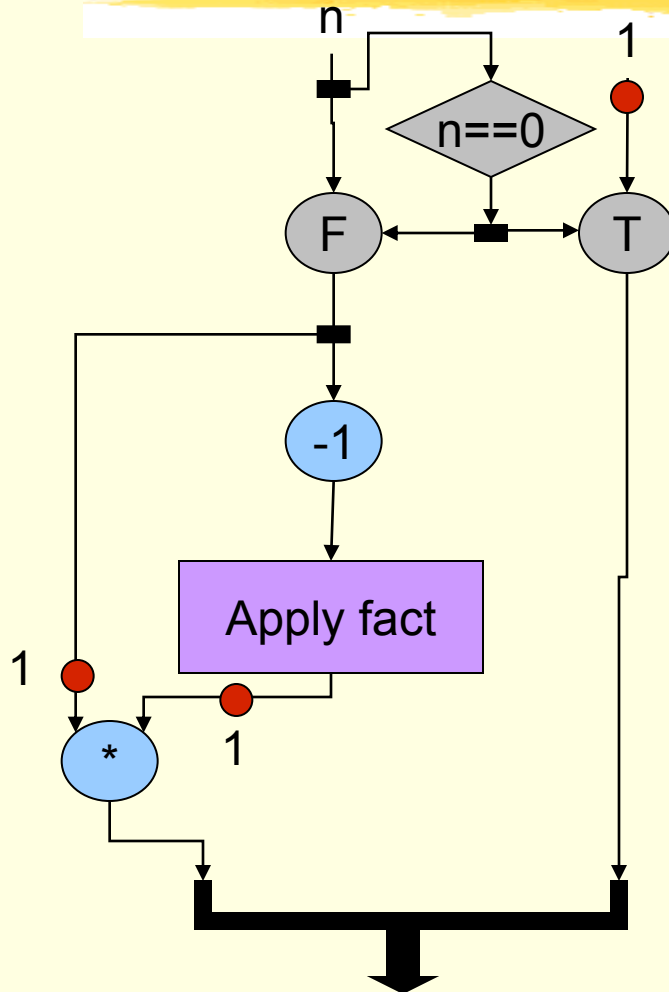


$3 * \text{fact}(2 * \text{fact}(1 * \text{fact}(0)))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

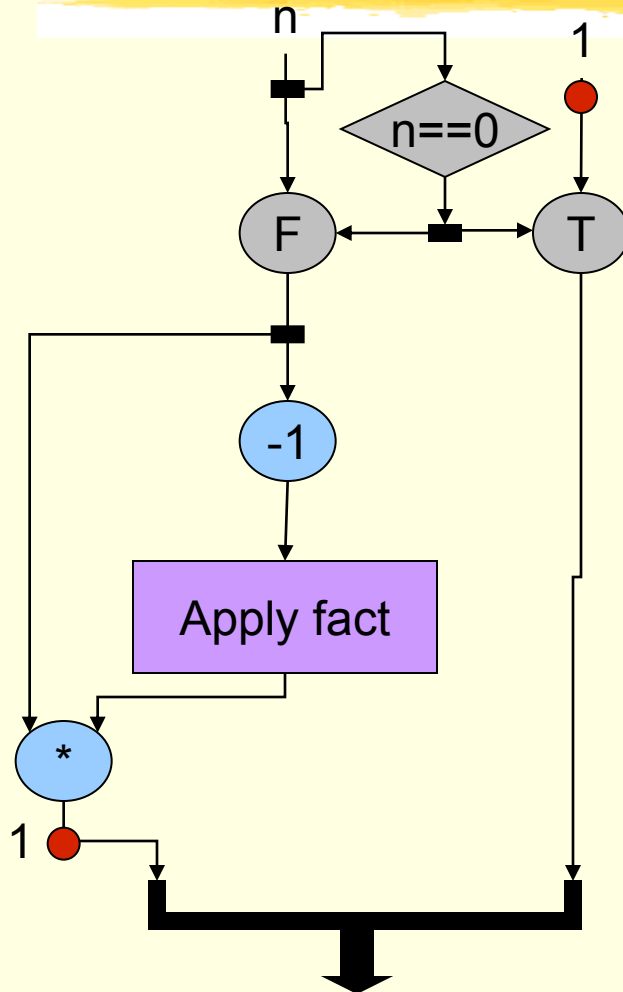


$3 * \text{fact}(2 * \text{fact}(1 * 1))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

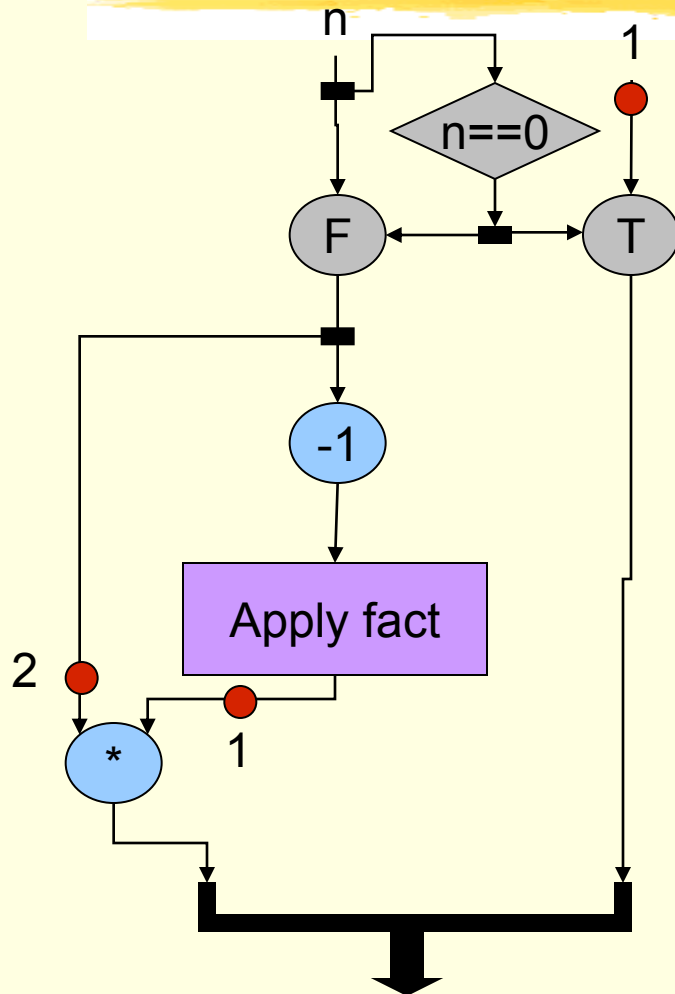


$3 * \text{fact}(2 * \text{fact}(1 * 1))$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

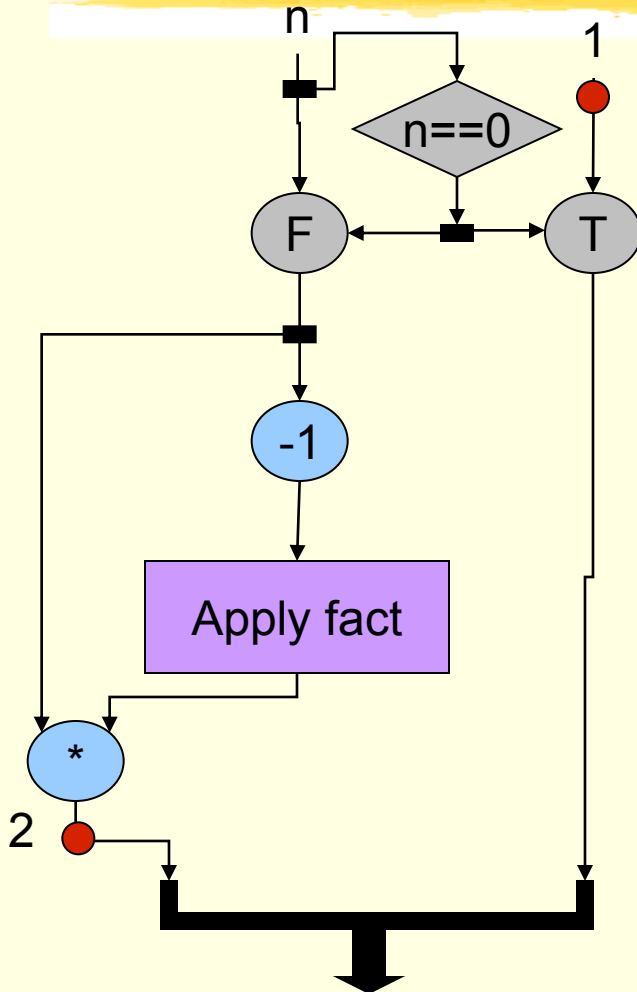


$3 * \text{fact}(2 * 1)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

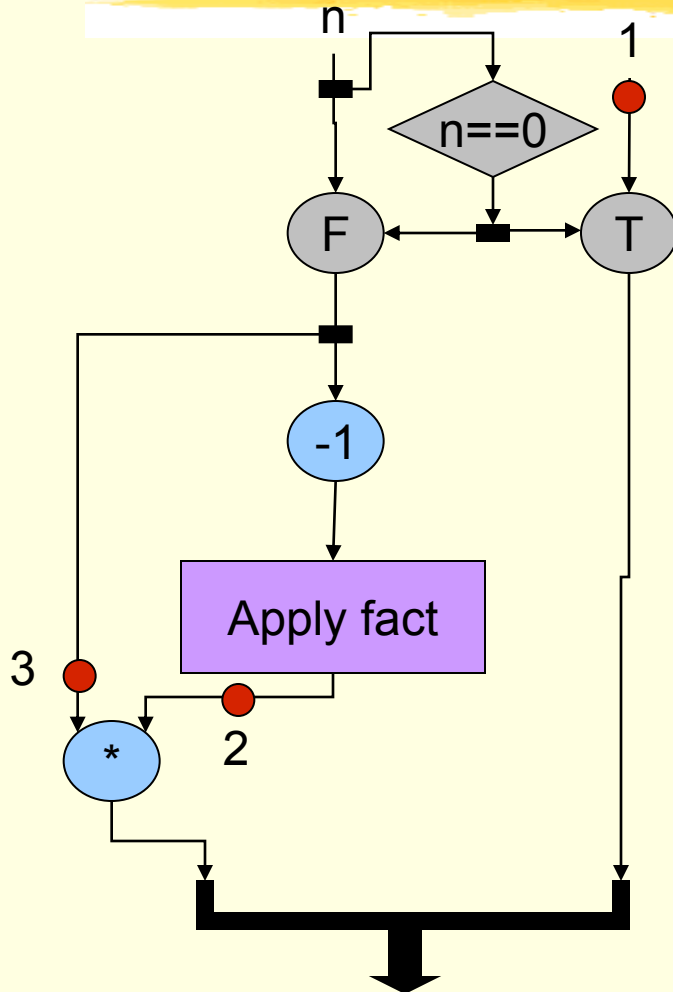


$3 * \text{fact}(2 * 1)$

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version

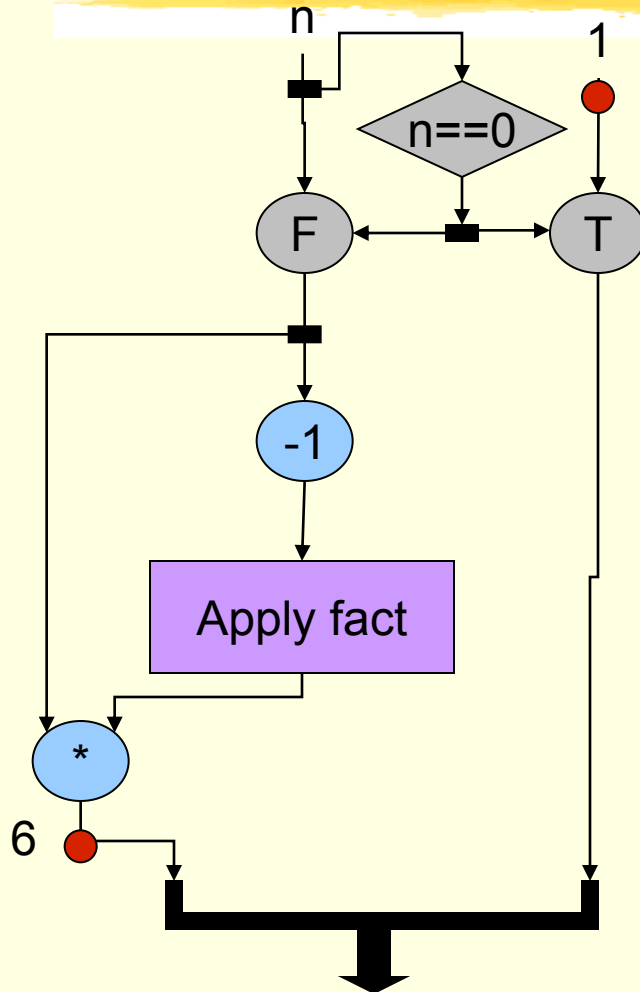


3 * 2

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

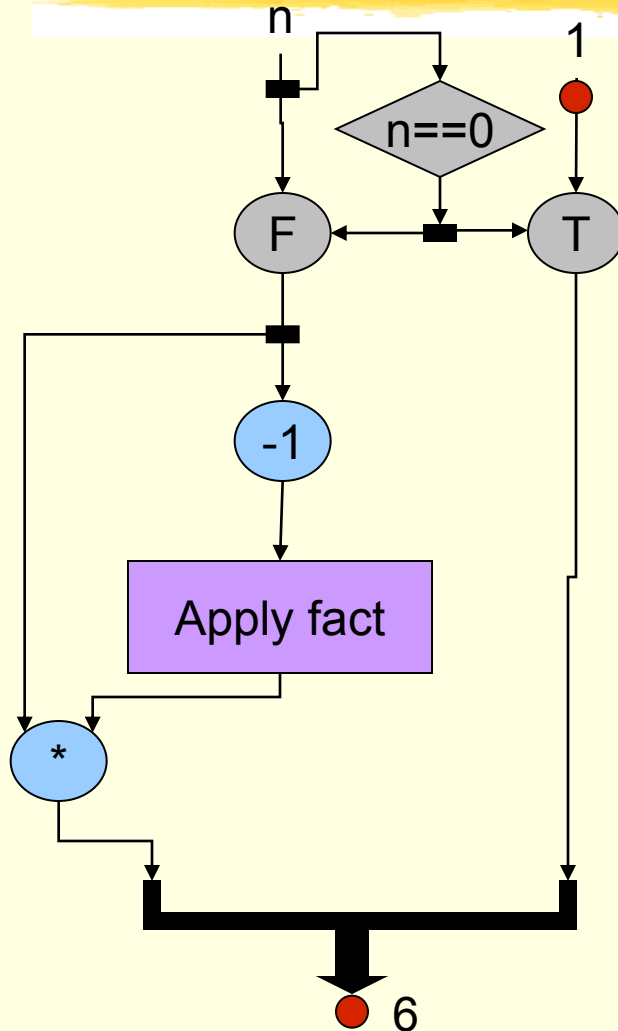
The Normal Version



```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Factorial

The Normal Version



6

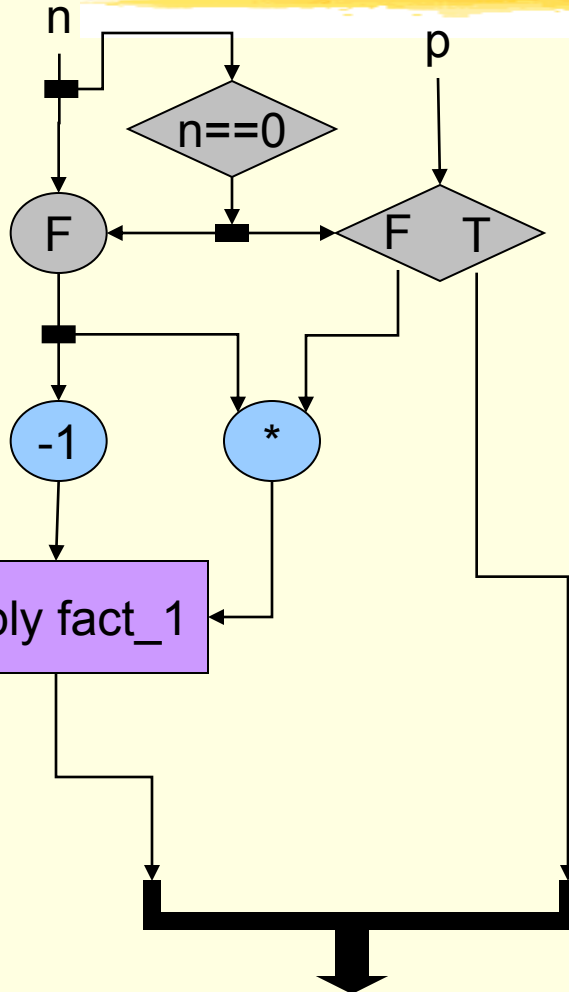
```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Can you see where is the problem ??



Factorial

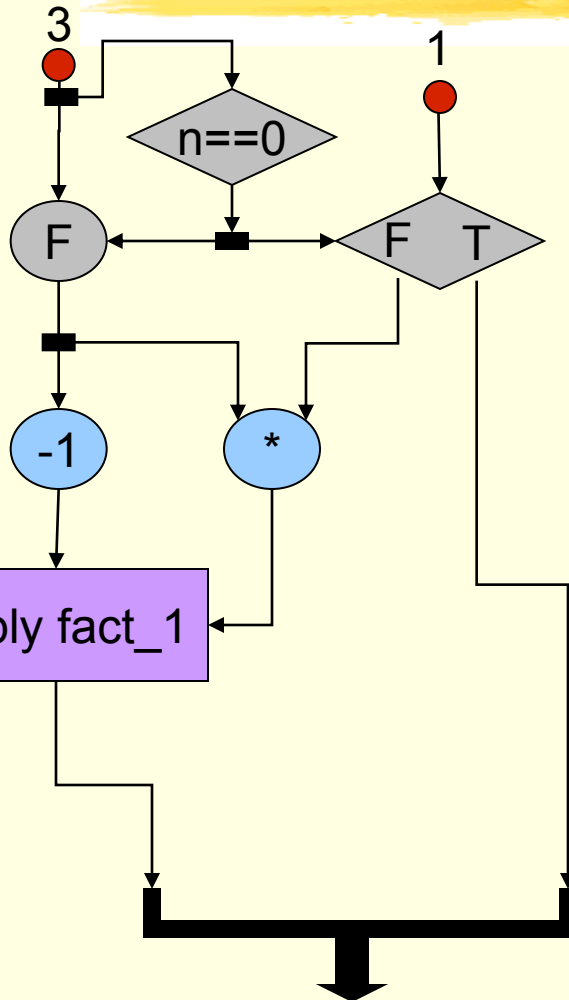
The Tail Recursion Version



```
long fact_1(n, p){
  if(n == 0) return p;
  else return fact_1(n-1, n*p);
}
```

Call fact_1(3,1)

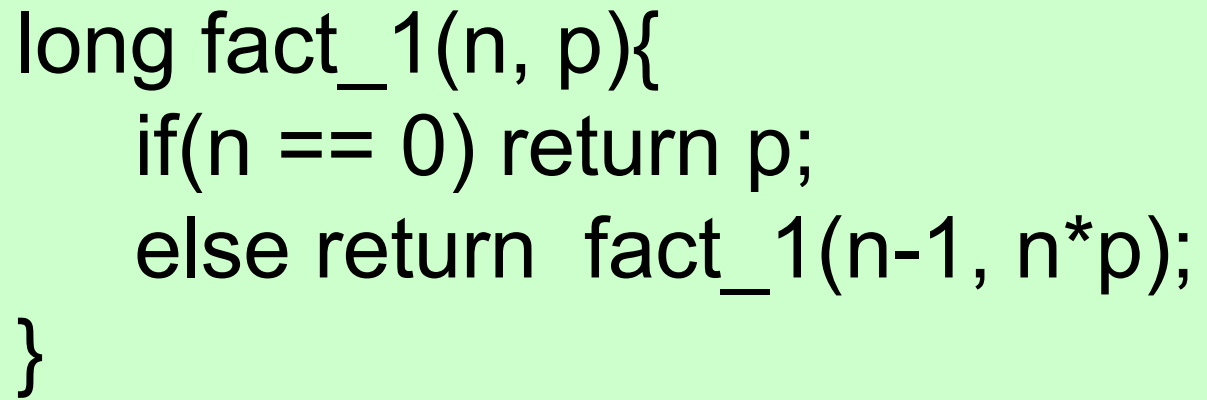
Factorial



```
long fact_1(n, p){
    if(n == 0) return p;
    else return fact_1(n-1, n*p);
}
```

fact_1(3,1)

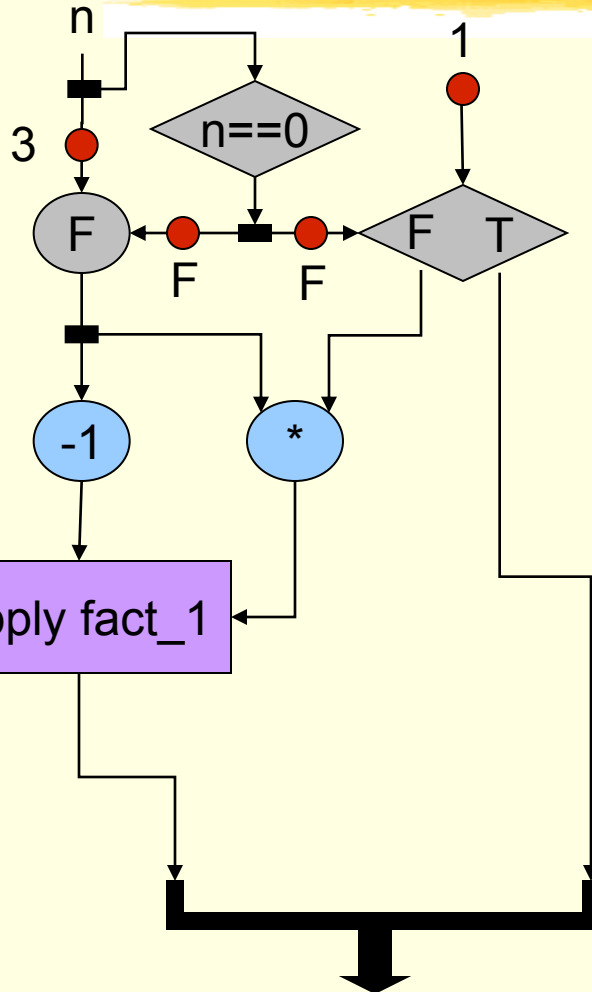
Factorial



fact_1(3,1)

Factorial

The Tail Recursion Version

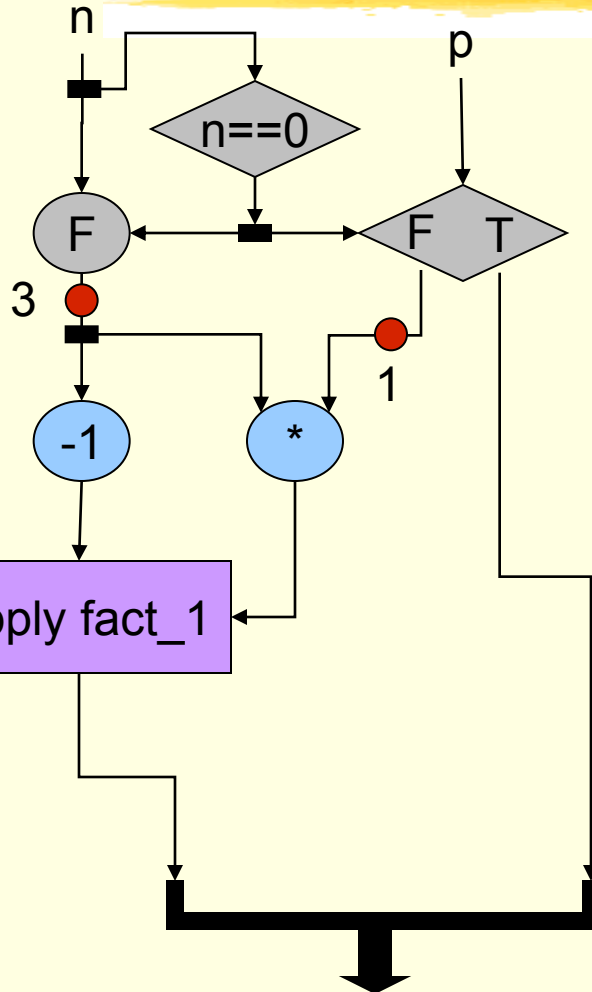


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(3,1)

Factorial

The Tail Recursion Version

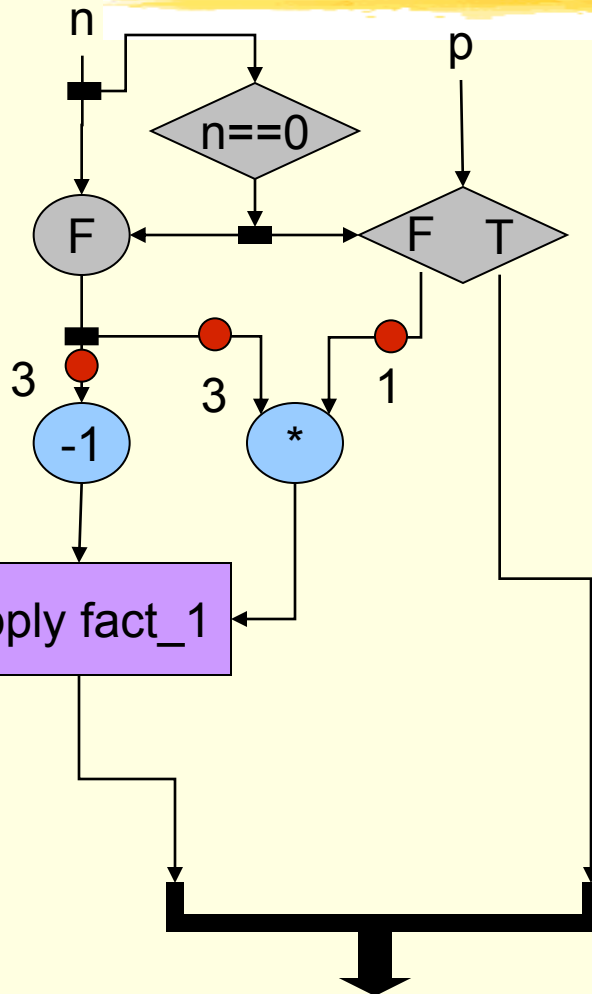


```
long fact_1(n, p){  
  if(n == 0) return p;  
  else return fact_1(n-1, n*p);  
}
```

fact_1(3,1)

Factorial

The Tail Recursion Version



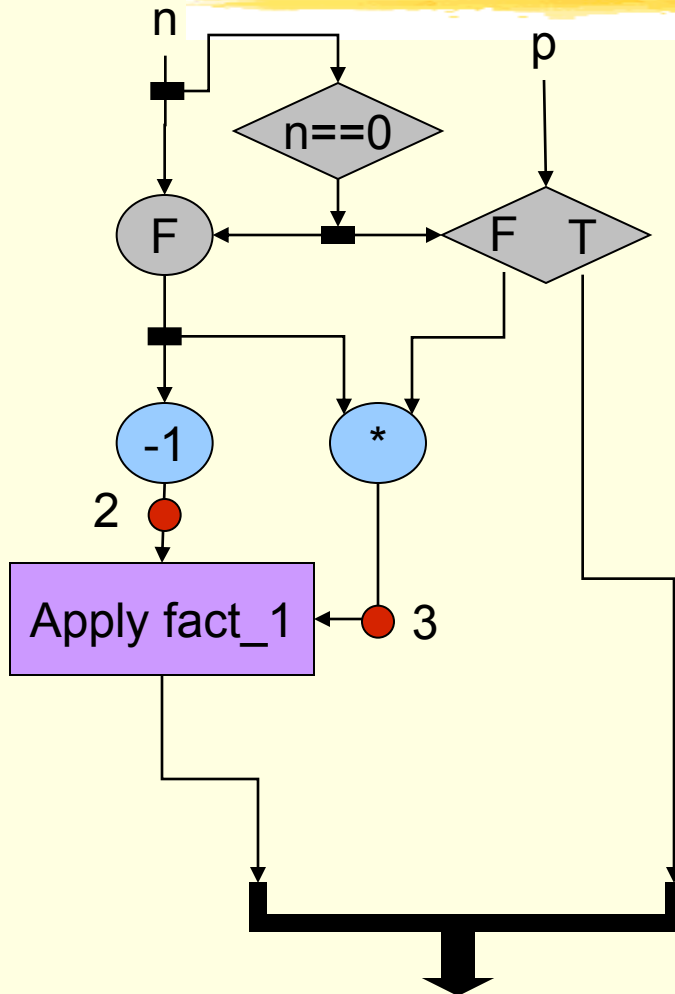
```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(3,1)

Factorial

The Tail Recursion Version

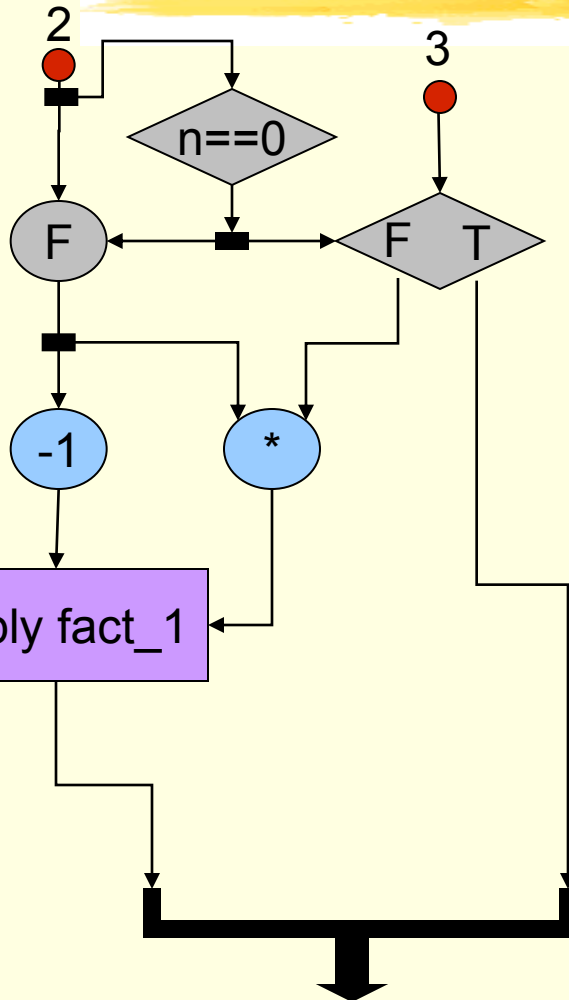
```
long fact_1(n, p){  
  if(n == 0) return p;  
  else return fact_1(n-1, n*p);  
}
```



fact_1(3,1)

Factorial

The Tail Recursion Version

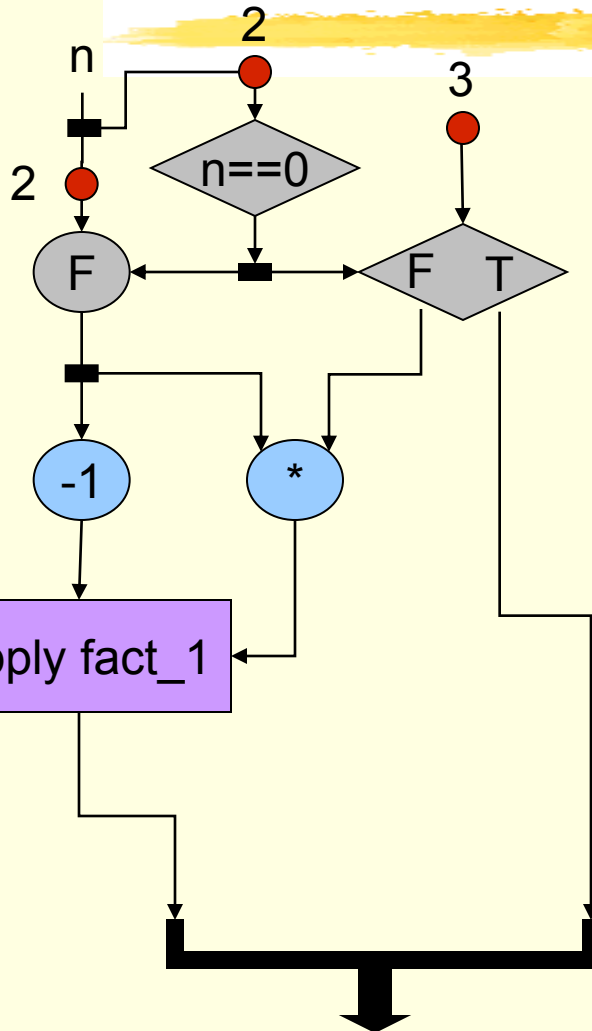


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

Call fact_1(2,3)

Factorial

The Tail Recursion Version

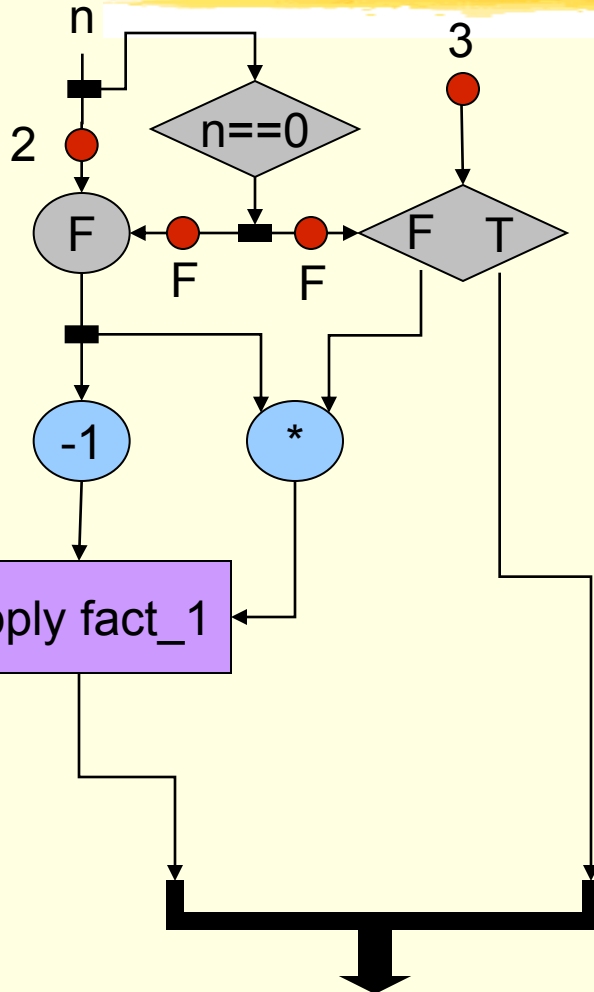


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(2,3)

Factorial

The Tail Recursion Version

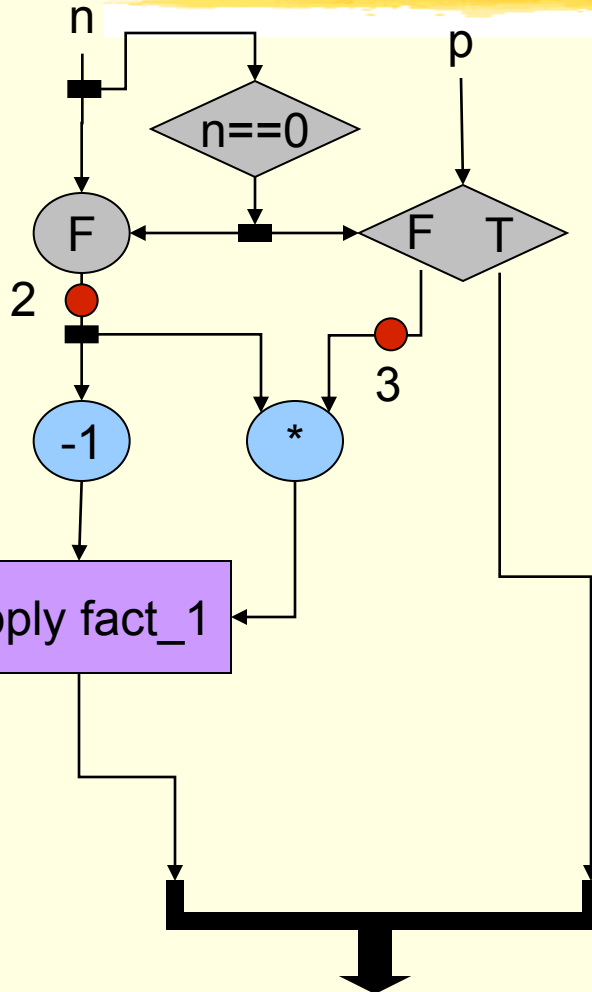


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(2,3)

Factorial

The Tail Recursion Version

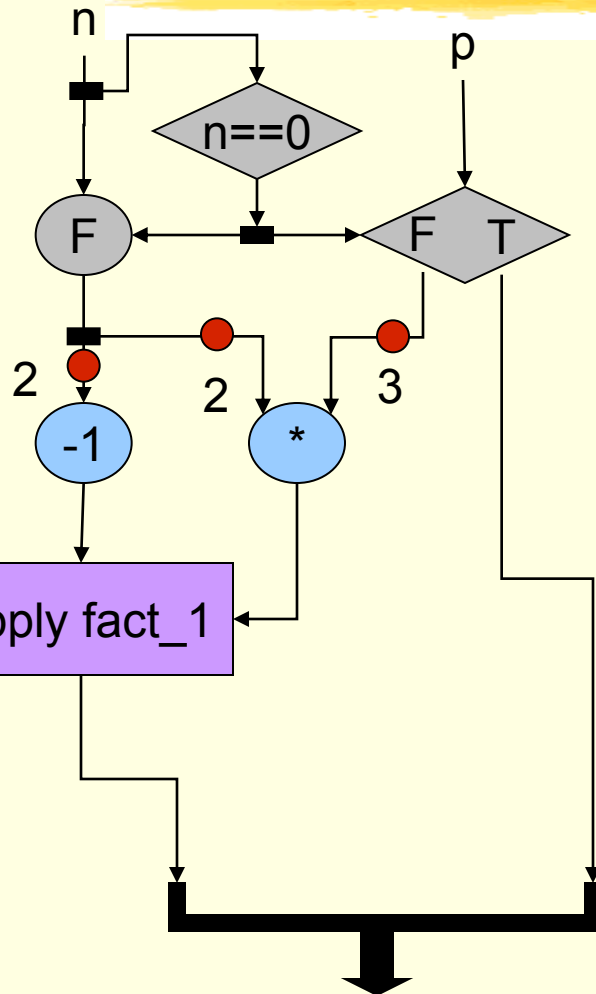


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(2,3)

Factorial

The Tail Recursion Version

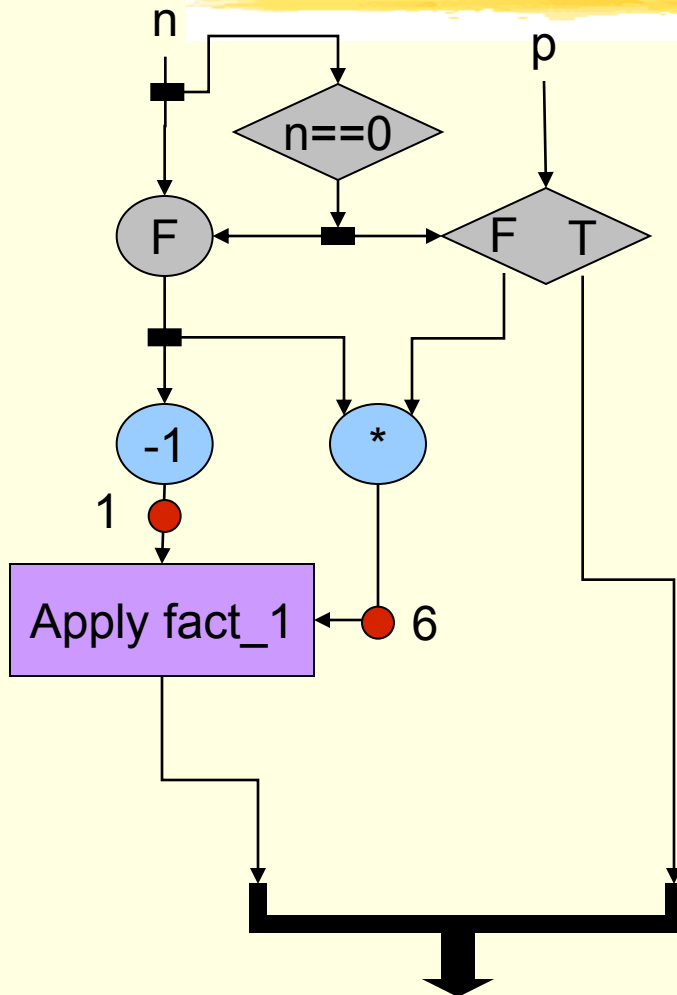


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(2,3)

Factorial

The Tail Recursion Version

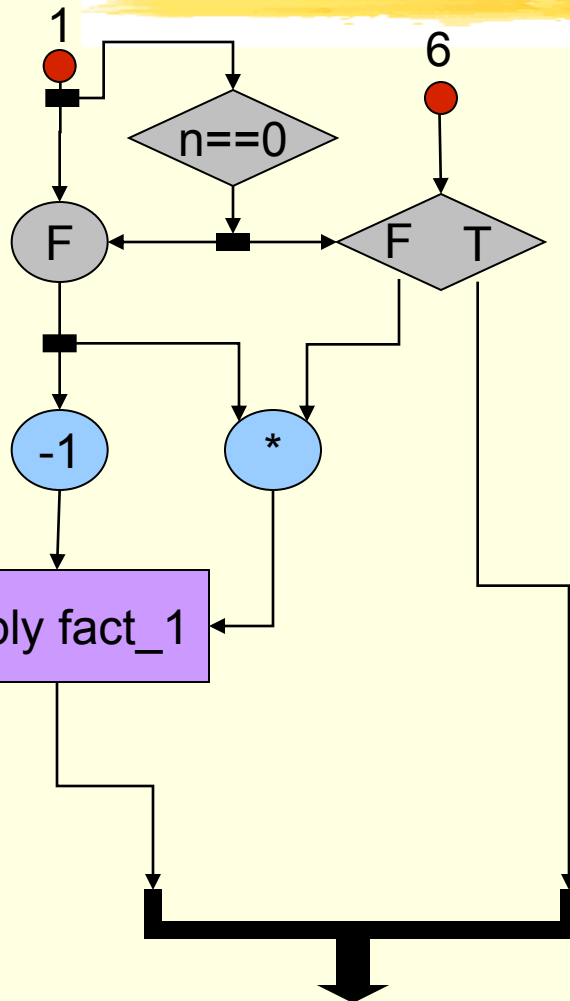


```
long fact_1(n, p){  
  if(n == 0) return p;  
  else return fact_1(n-1, n*p);  
}
```

Call fact_1 (1,6)

Factorial

The Tail Recursion Version

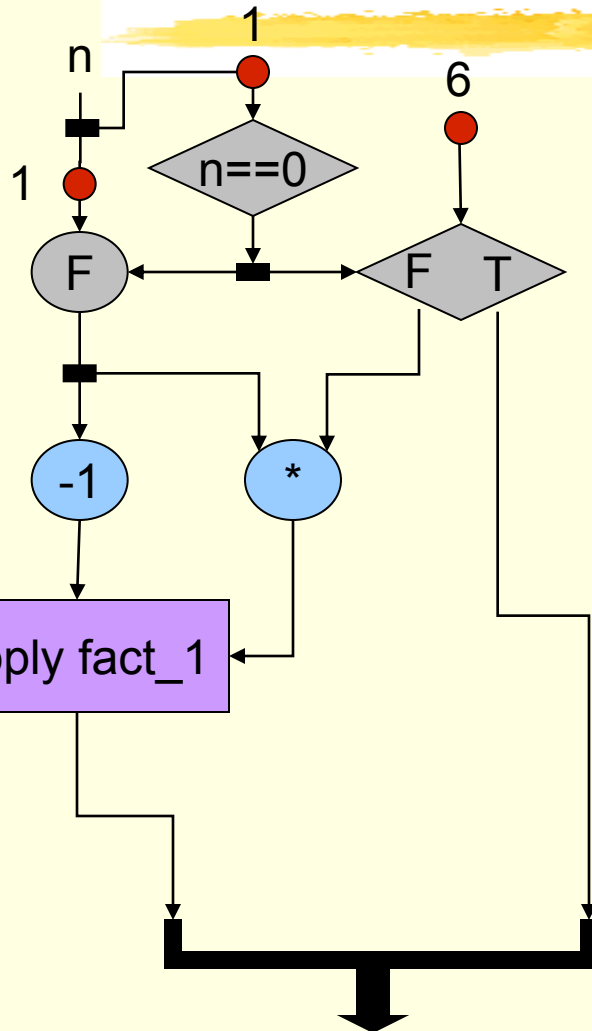


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(1,6)

Factorial

The Tail Recursion Version

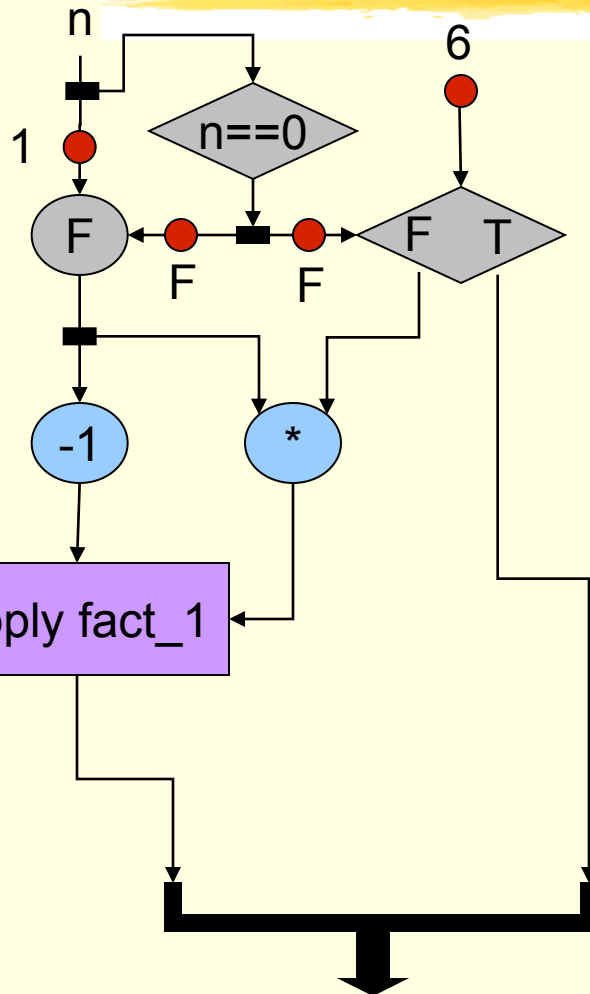


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(1,6)

Factorial

The Tail Recursion Version

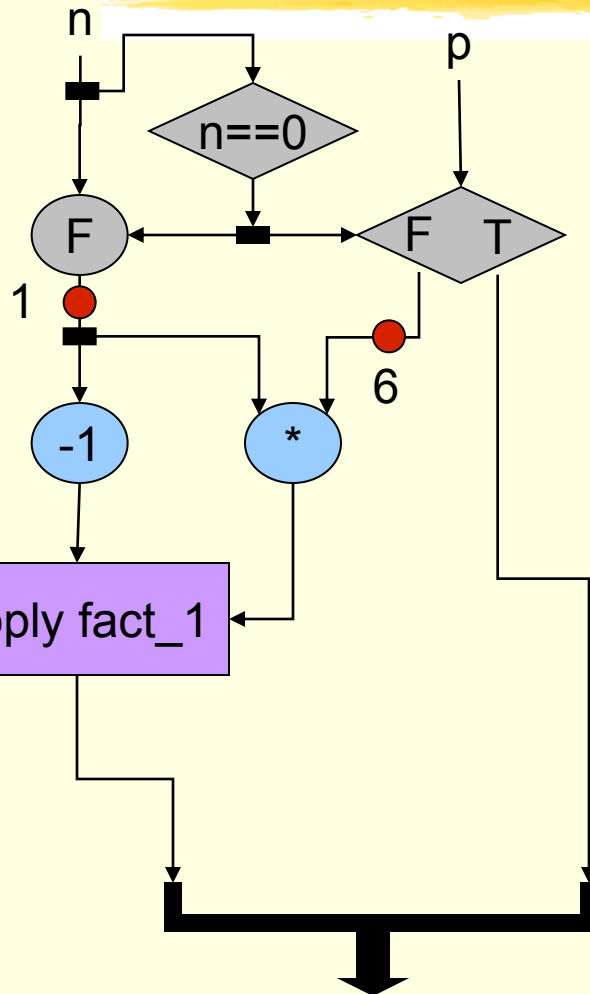


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(1,6)

Factorial

The Tail Recursion Version

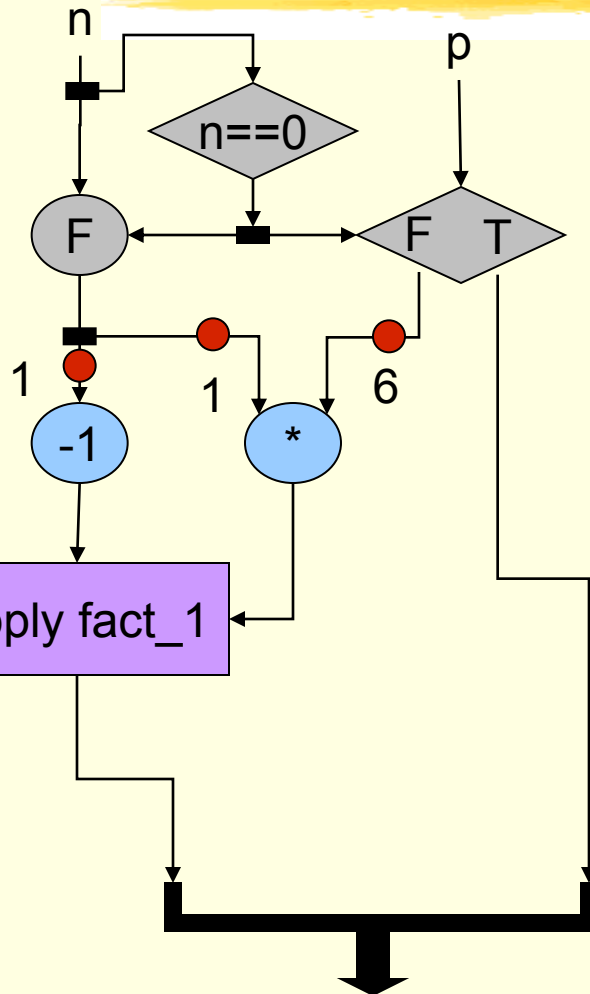


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(1,6)

Factorial

The Tail Recursion Version

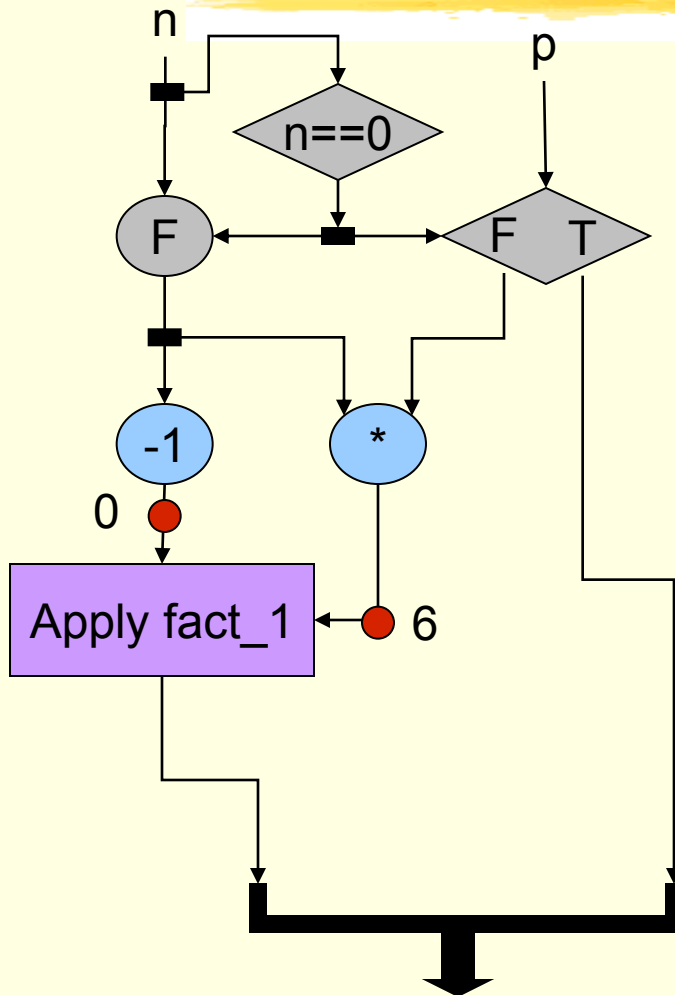


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(1,6)

Factorial

The Tail Recursion Version

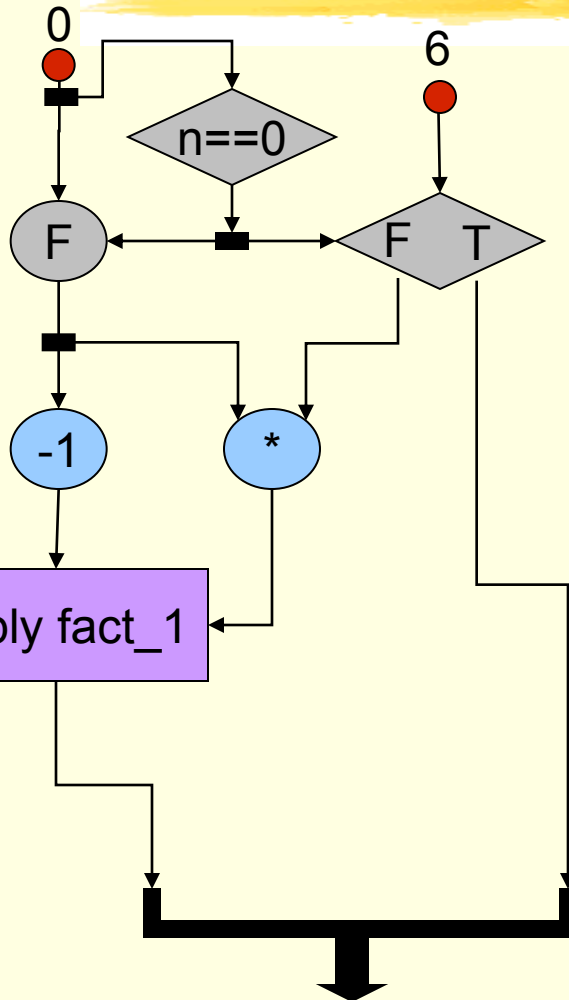


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(1,6)

Factorial

The Tail Recursion Version

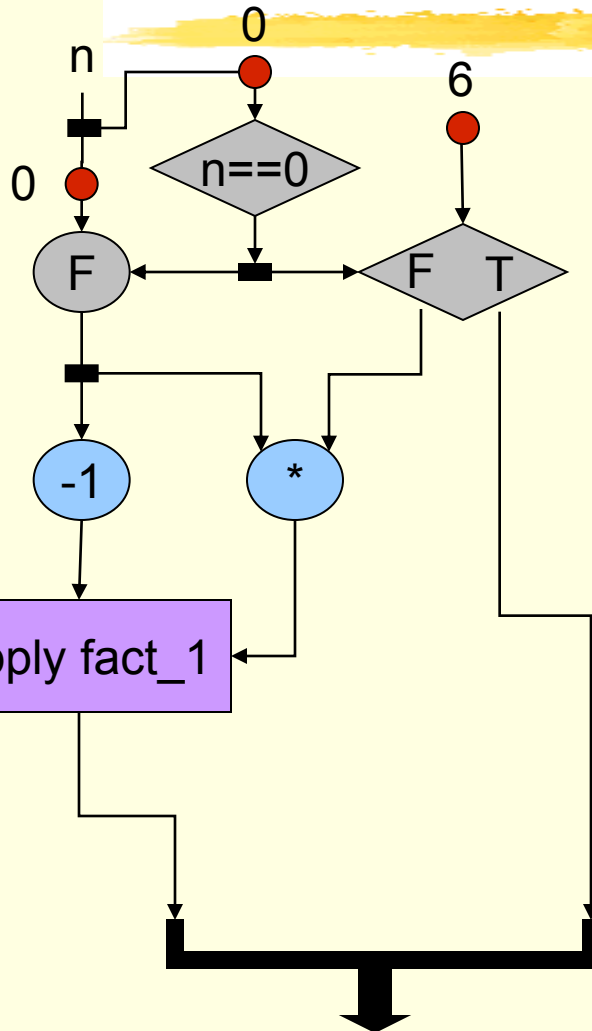


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

Call fact(0,6)

Factorial

The Tail Recursion Version

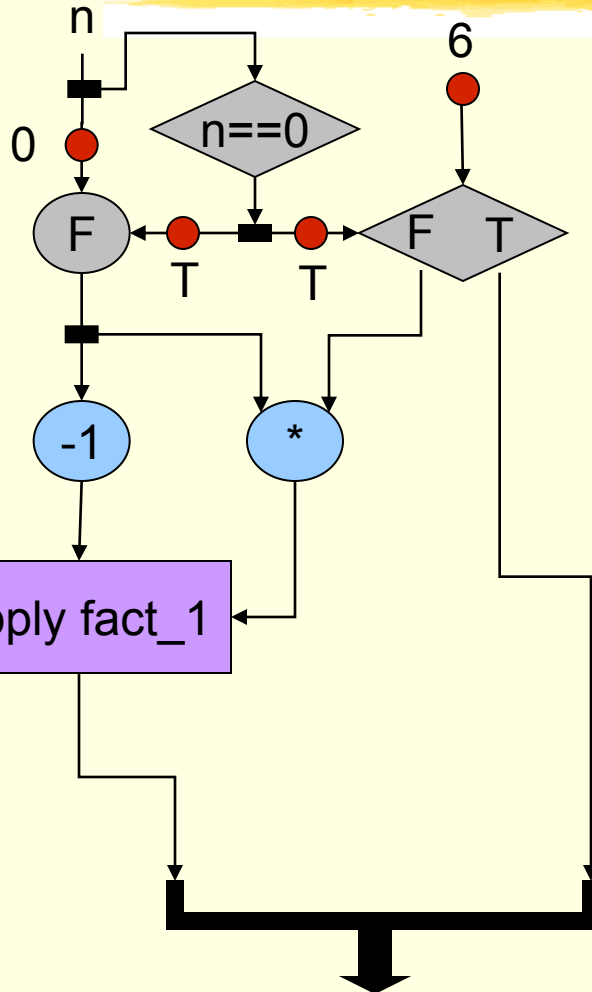


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(0,6)

Factorial

The Tail Recursion Version

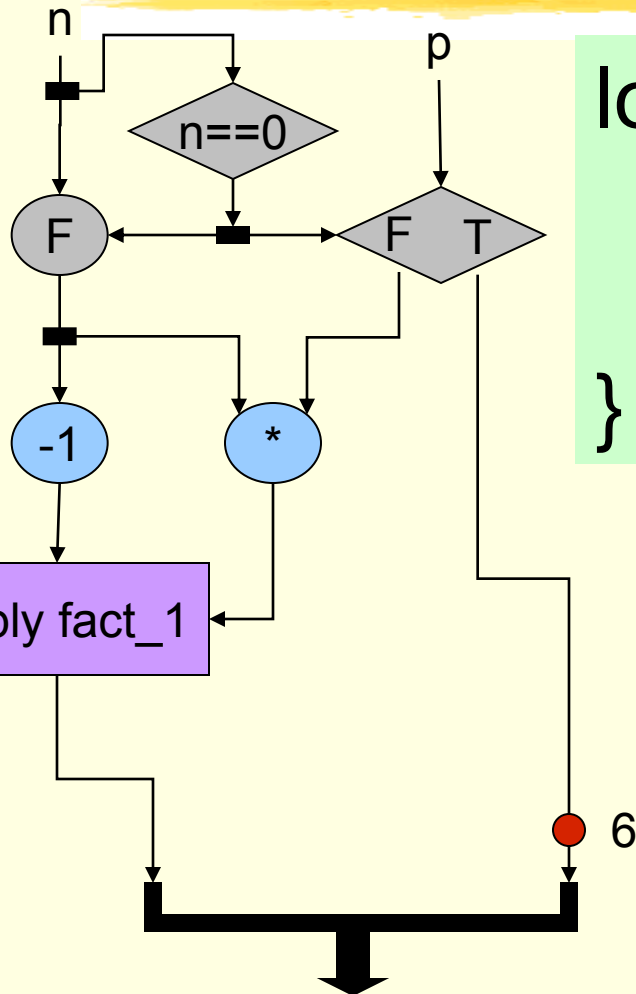


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(0,6)

Factorial

The Tail Recursion Version

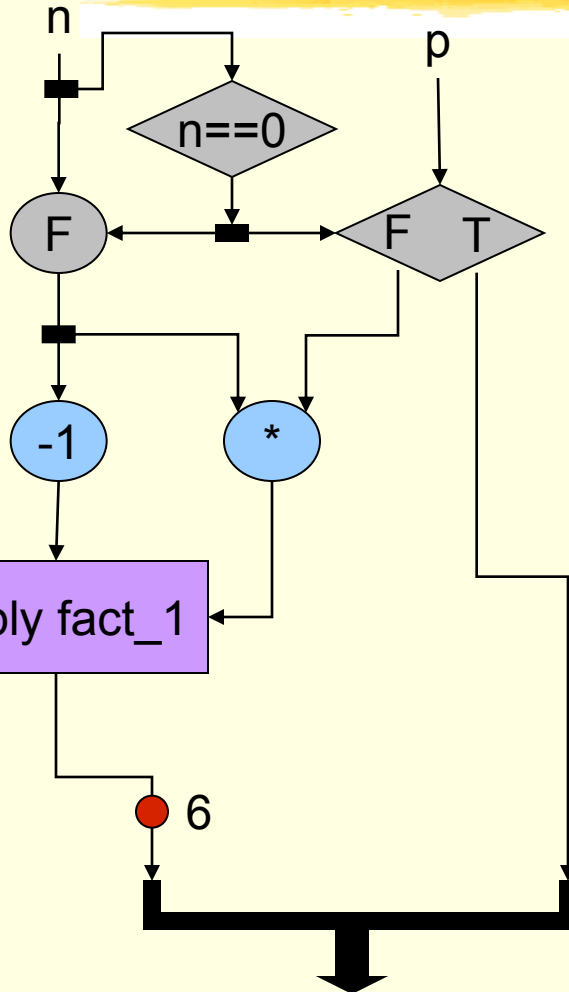


```
long fact_1(n, p){  
  if(n == 0) return p;  
  else return fact_1(n-1, n*p);  
}
```

fact_1(0,6) returns 6

Factorial

The Tail Recursion Version

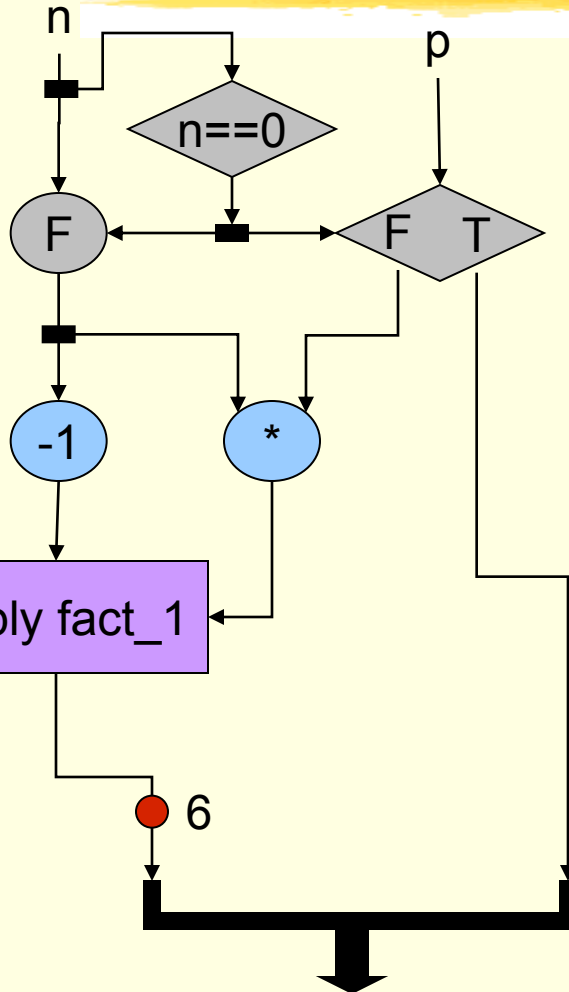


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(1,6) returns 6

Factorial

The Tail Recursion Version

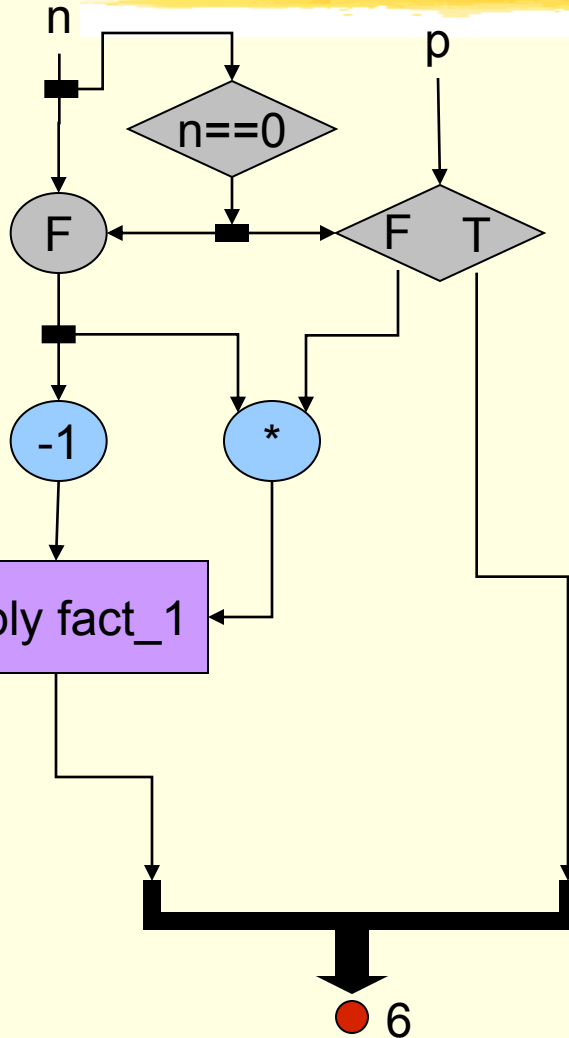


```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

fact_1(3,1) returns 6

Factorial

The Tail Recursion Version



```
long fact_1(n, p){  
    if(n == 0) return p;  
    else return fact_1(n-1, n*p);  
}
```

6

Recursive Program Graph Features



- Acyclic
- One-token-per-link-in-lifetime
- Tags
- No deterministic merge needed
- Recursion is expressed by runtime copying
- No matching is needed (why?)

A Quiz: $y = x^n$?



A Recursive Version of Power (not tail recursive)

Function Rec-Power (x, n : integer
returns integer)

Returns

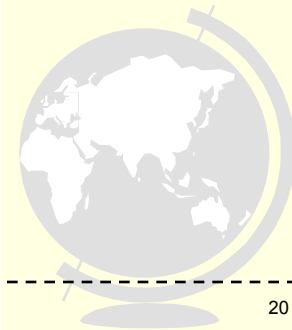
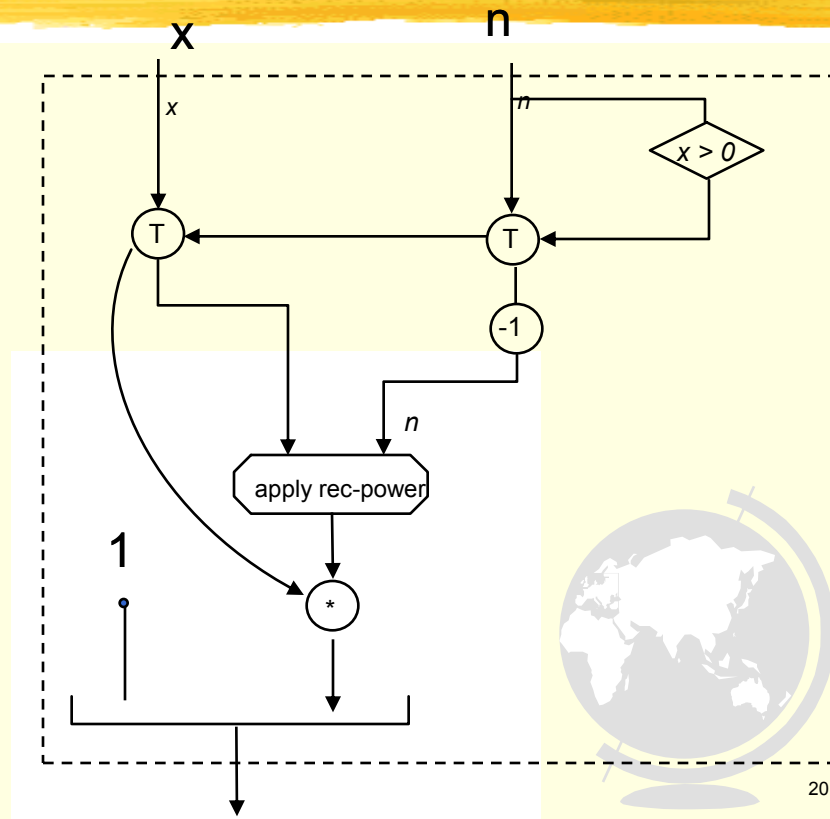
If $n = 0$ then 1

else

$x * \text{rec-power}(x, n - 1)$

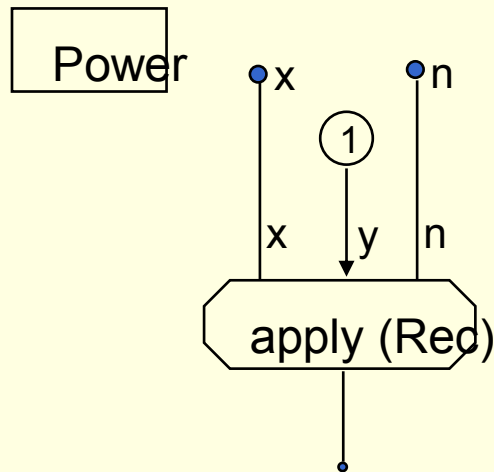
endif

endfun

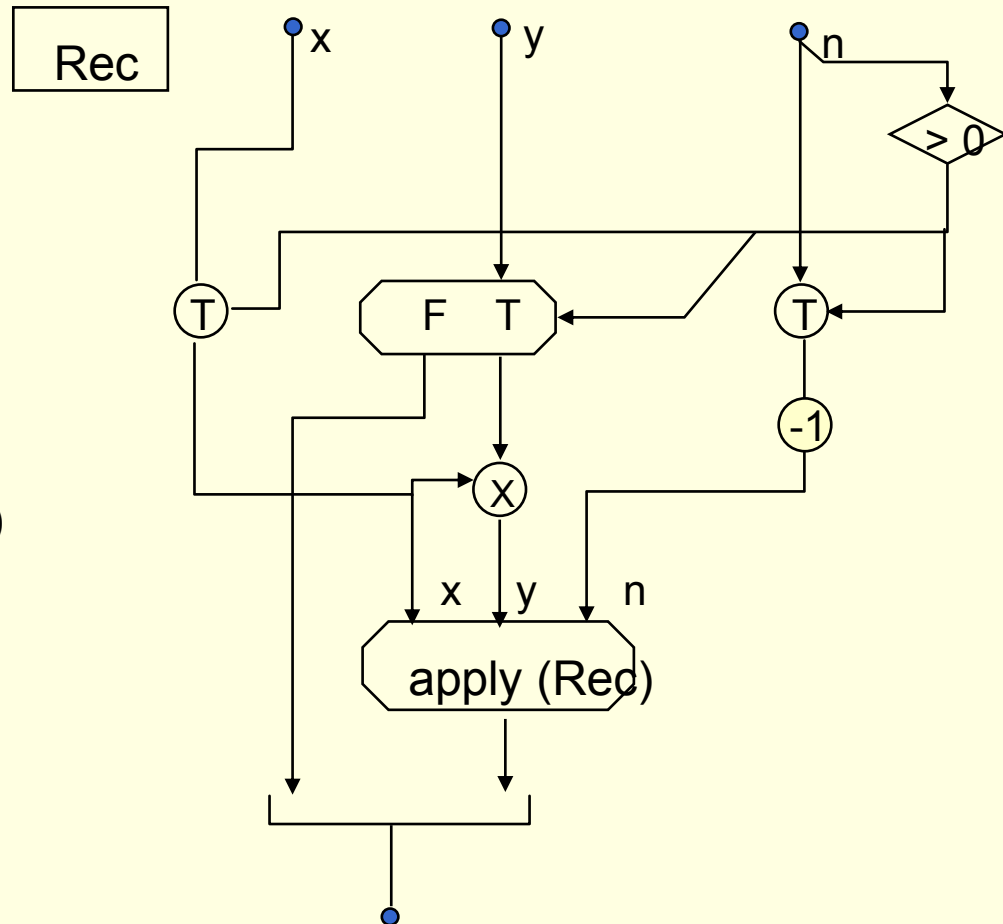


**Note: tail-recursive = iterations:
i.e. the states of the computation are captured
explicitly by the set of iteration variables.**

The power function as a recursive program graph



```
function Rec (x,y,n)
if n = 0 then y
else
  Rec (x, x*y, n-1)
endif
end function
Rec (x,1,n)
```



Dynamic Dataflow



- Static Dataflow

Only one token per arc

Problems with Function calls, nested loops and data structures

A signal is needed to allow the parent's operator to fire

- Dynamic Dataflow

No limitations on number of tokens per arc

Tokens have tags – new firing rules and tag matching

The MIT tagged token dataflow model

Dynamic Dataflow



- Loops and function calls
Should be executed in parallel as instances of the same graph
- Arc → a container with different tokens that have different tags
- A node can fire as soon that all tokens with identical tags are presented in its input arcs

Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)

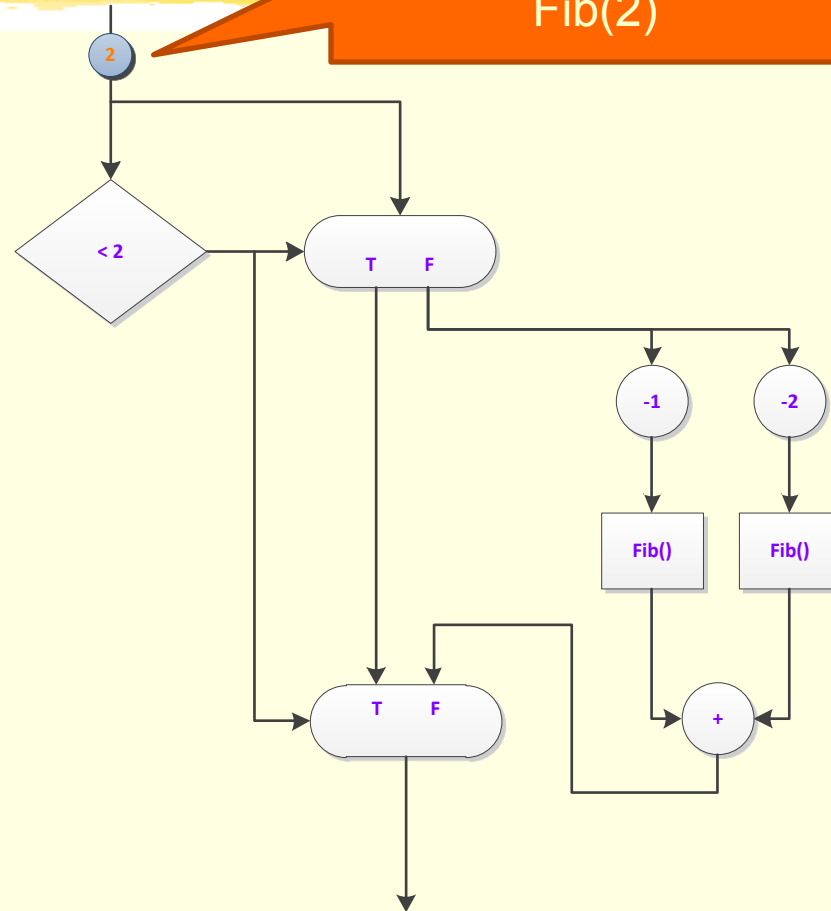


- On recursion a new color is assigned via a special node called the apply function
- On return the original color is restored

Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwe

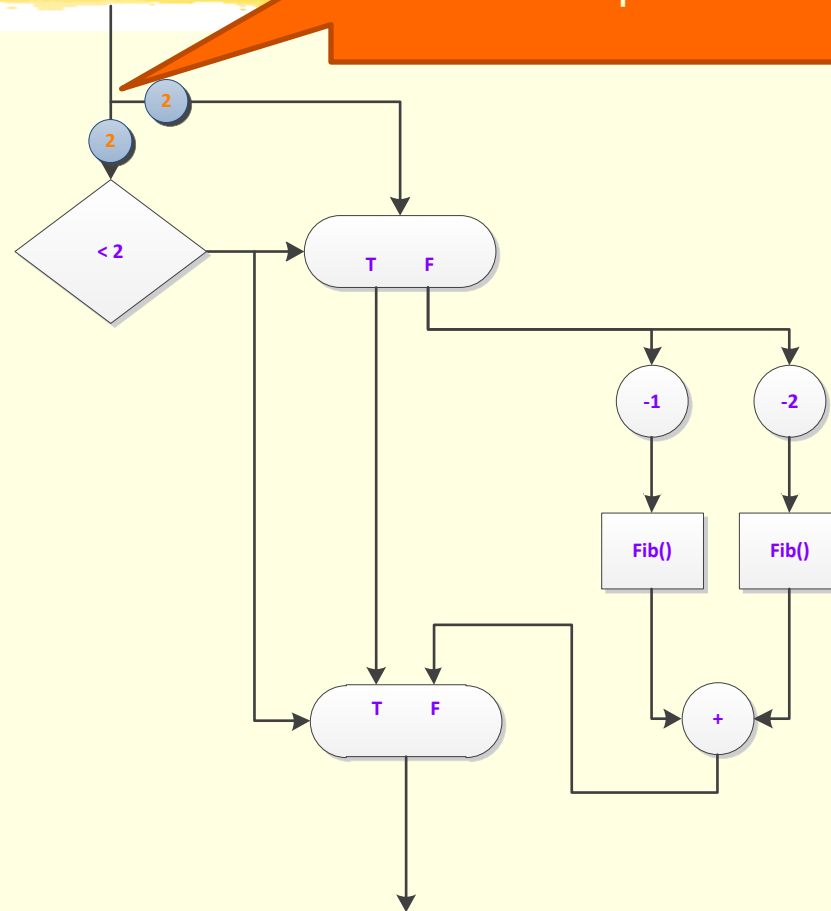
Let's begin: we have 1 token inserted for Fib(2)



Dynamic Dataflow: Colored Tokens

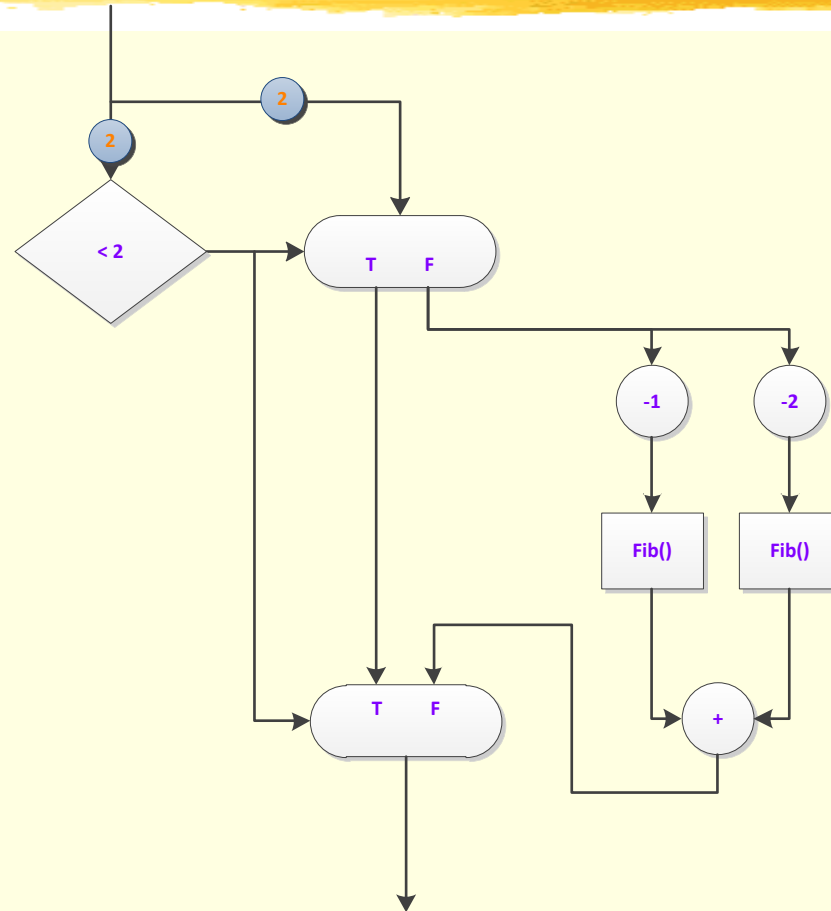
(Thanks to J.Landwe

It splits into two token here at a split node



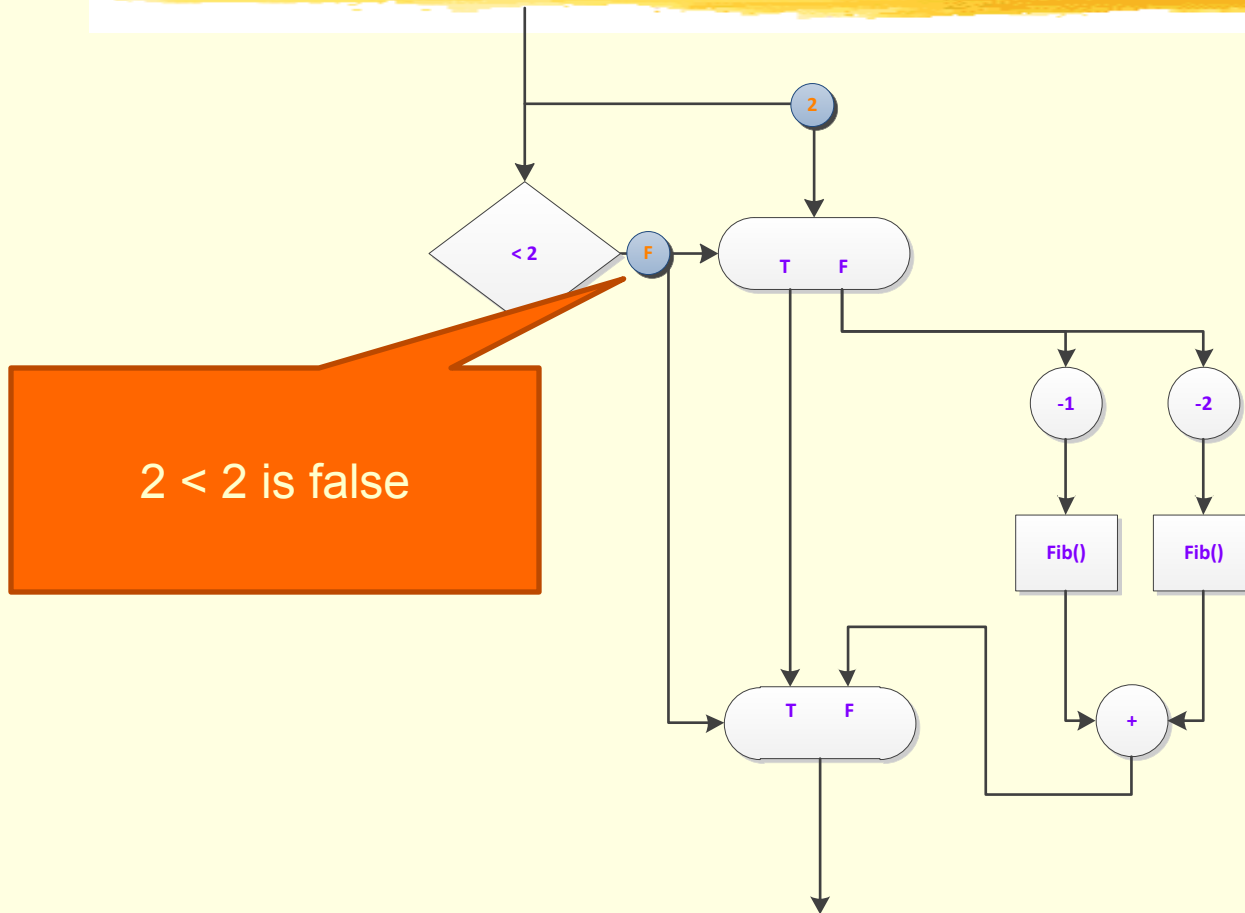
Dynamic Dataflow: Colored Tokens

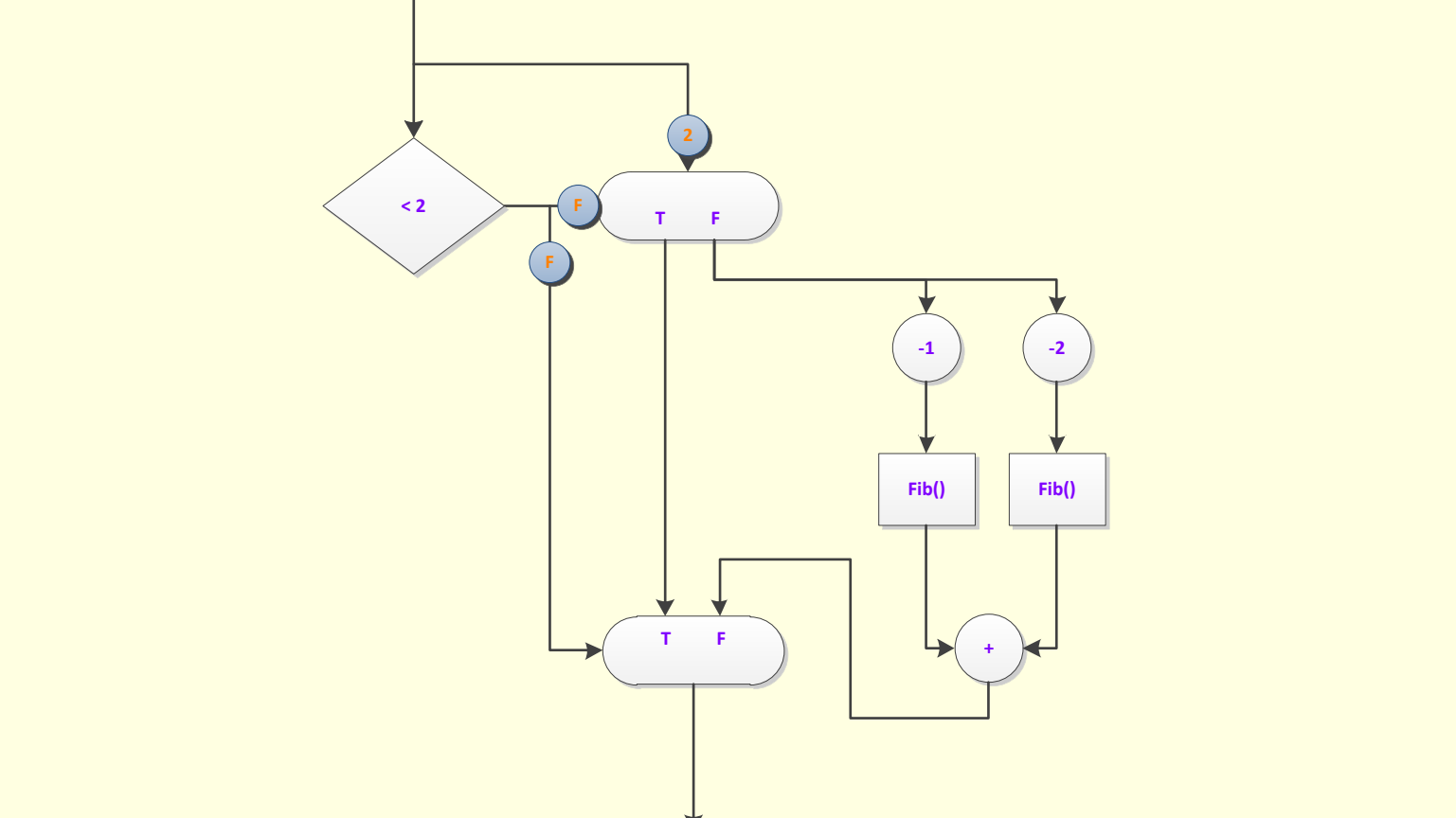
(Thanks to J.Landwehr)



Dynamic Dataflow: Colored Tokens

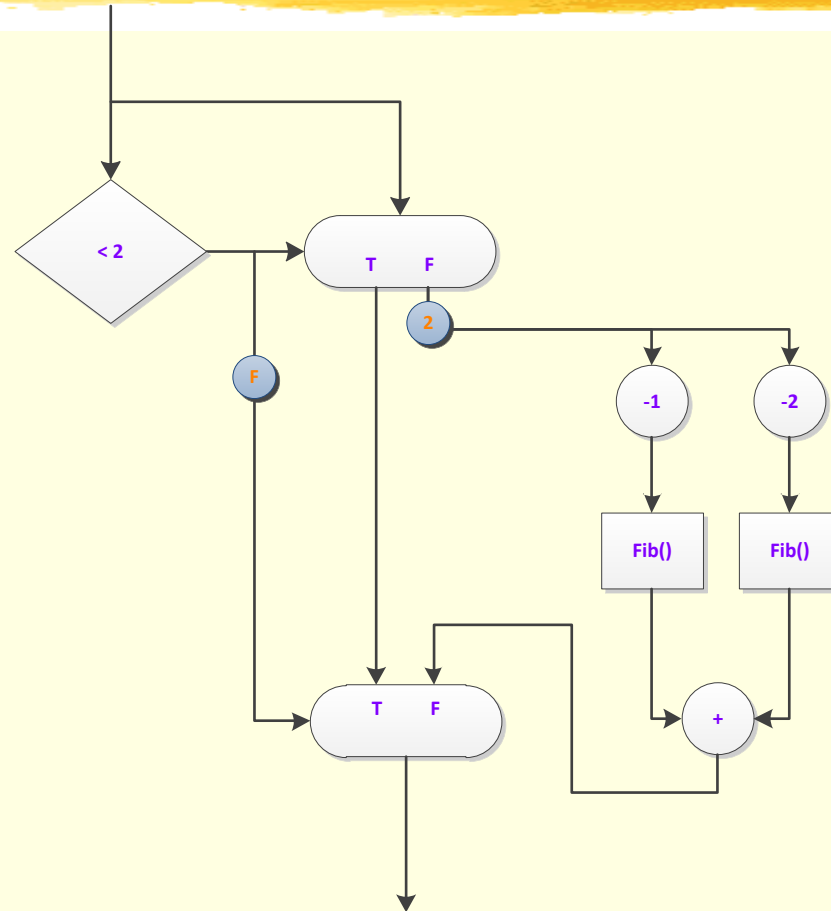
(Thanks to J.Landwehr)





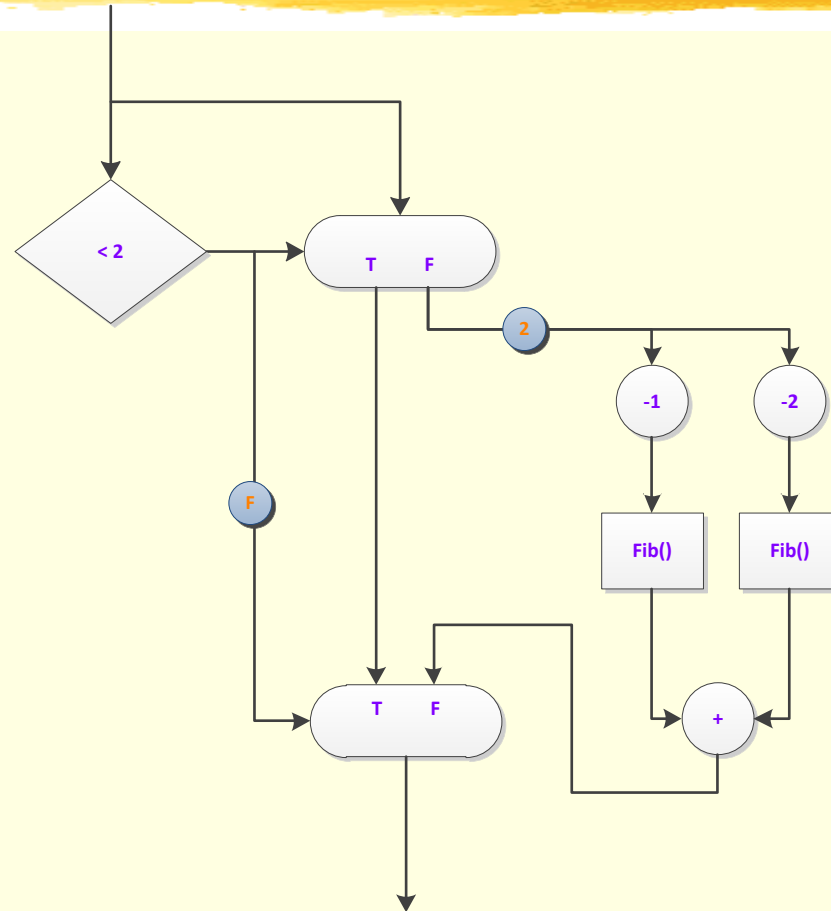
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



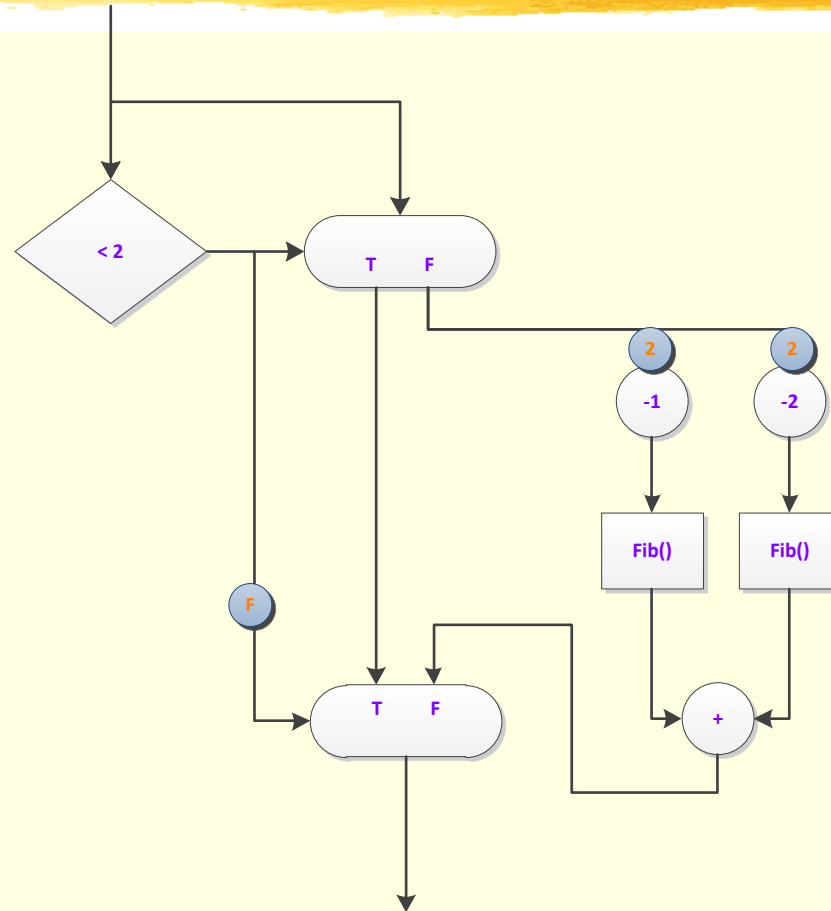
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



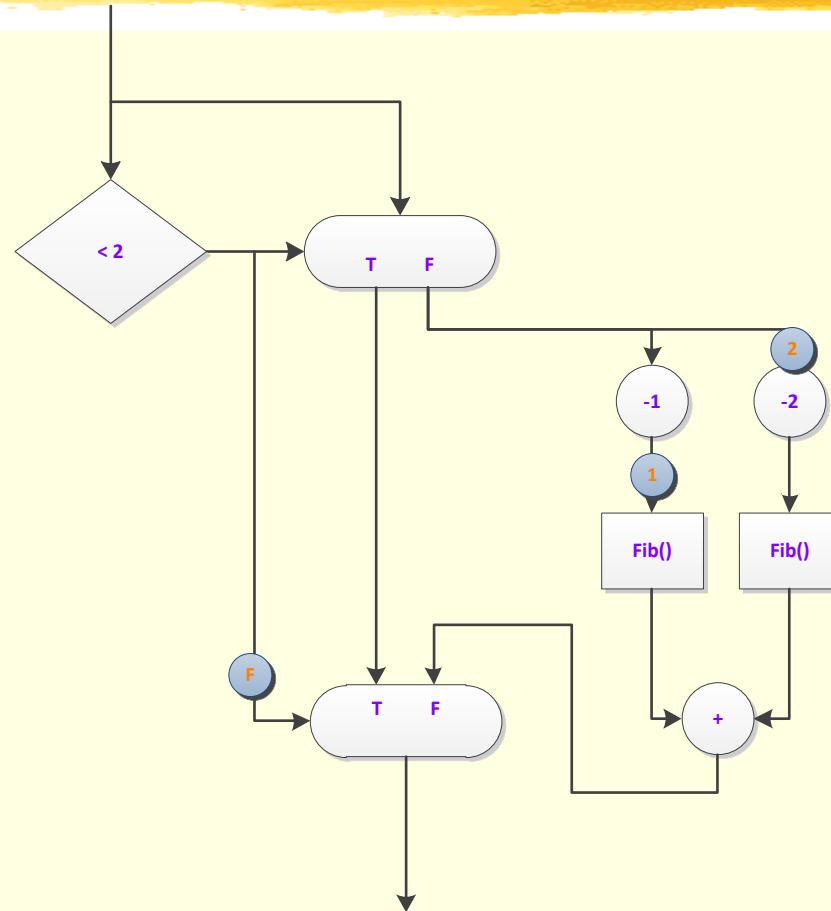
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



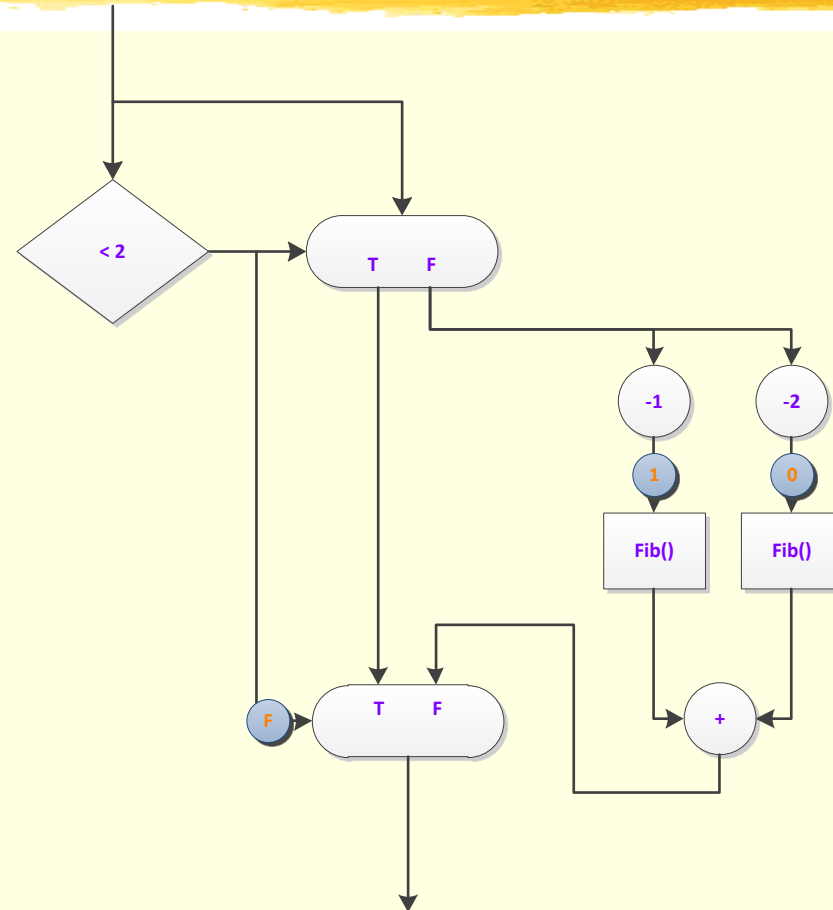
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



Dynamic Dataflow: Colored Tokens

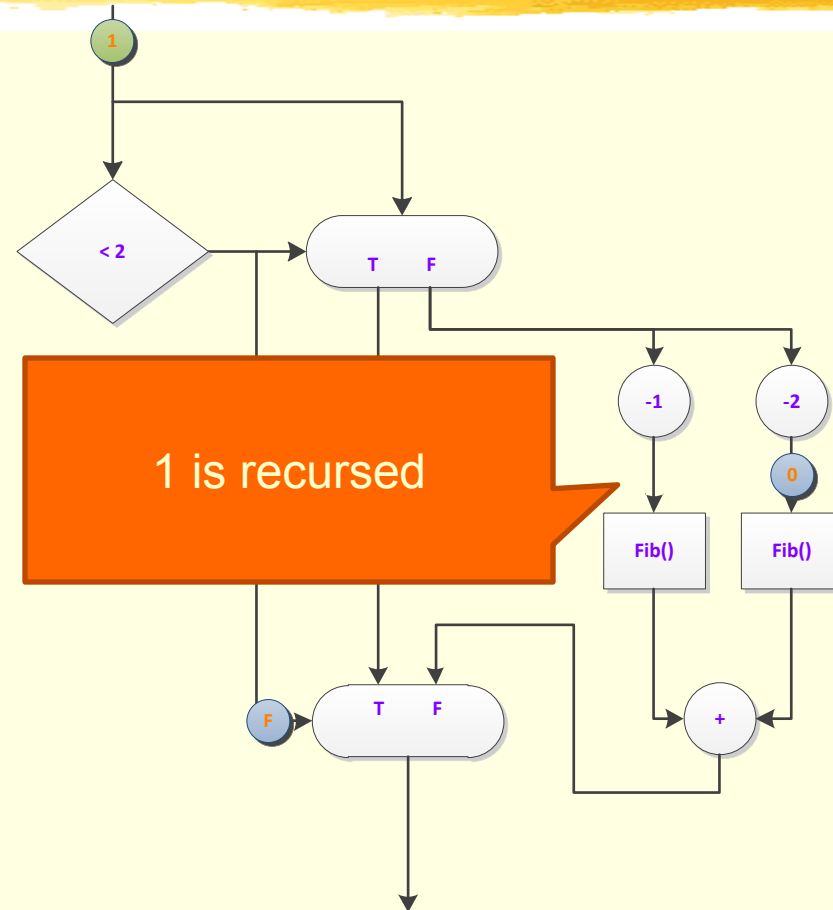
(Thanks to J.Landwehr)



Recursively call the
Fib graph

Dynamic Dataflow: Colored Tokens

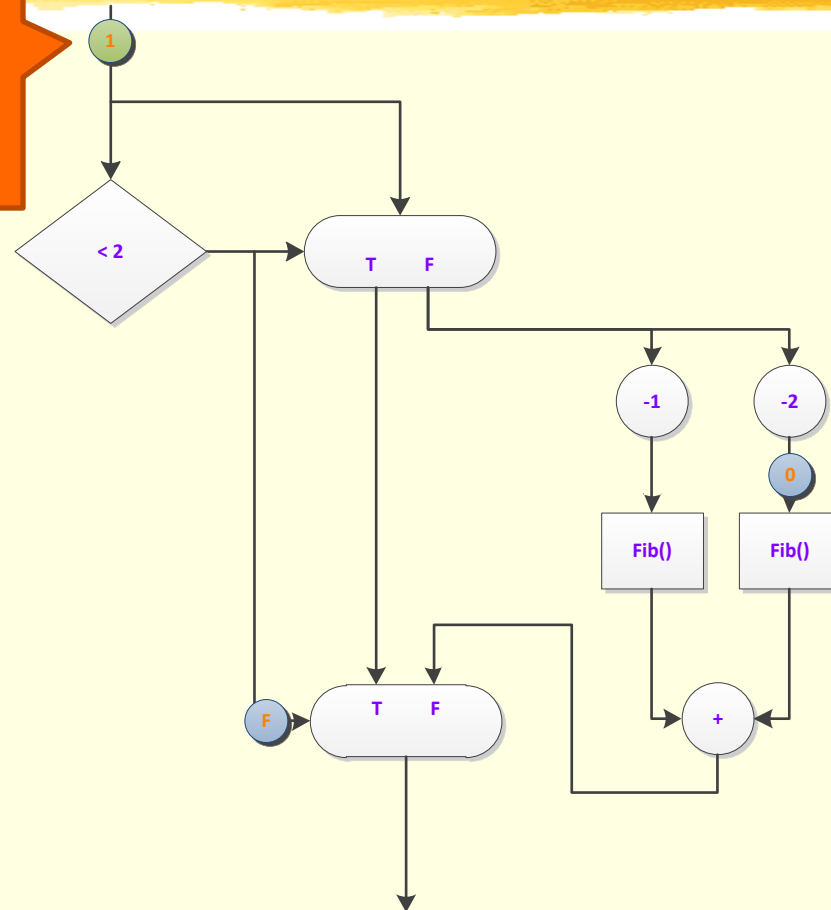
(Thanks to J.Landwehr)



Dynamic Dataflow: Colored Tokens

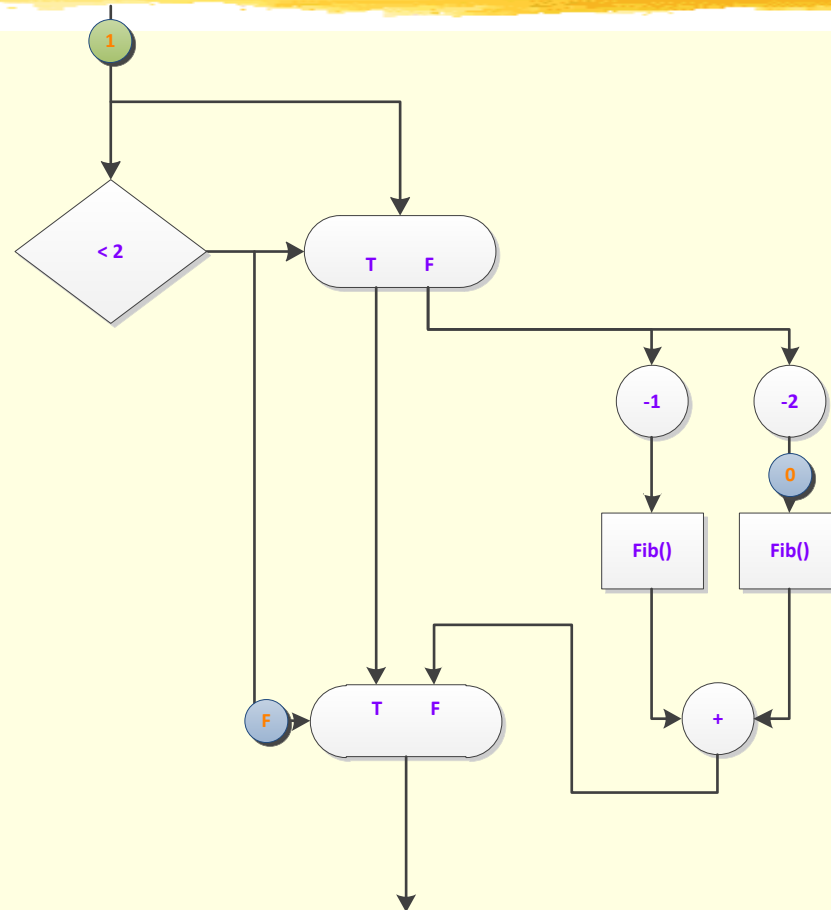
(Thanks to J.Landwehr)

New color chosen on recursion



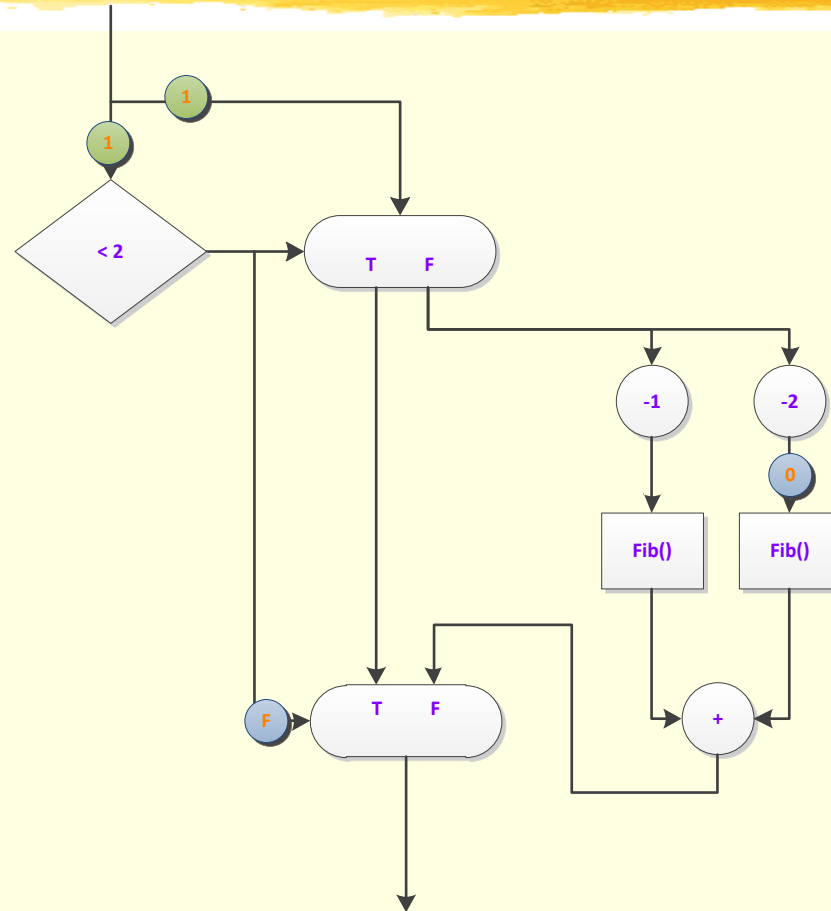
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



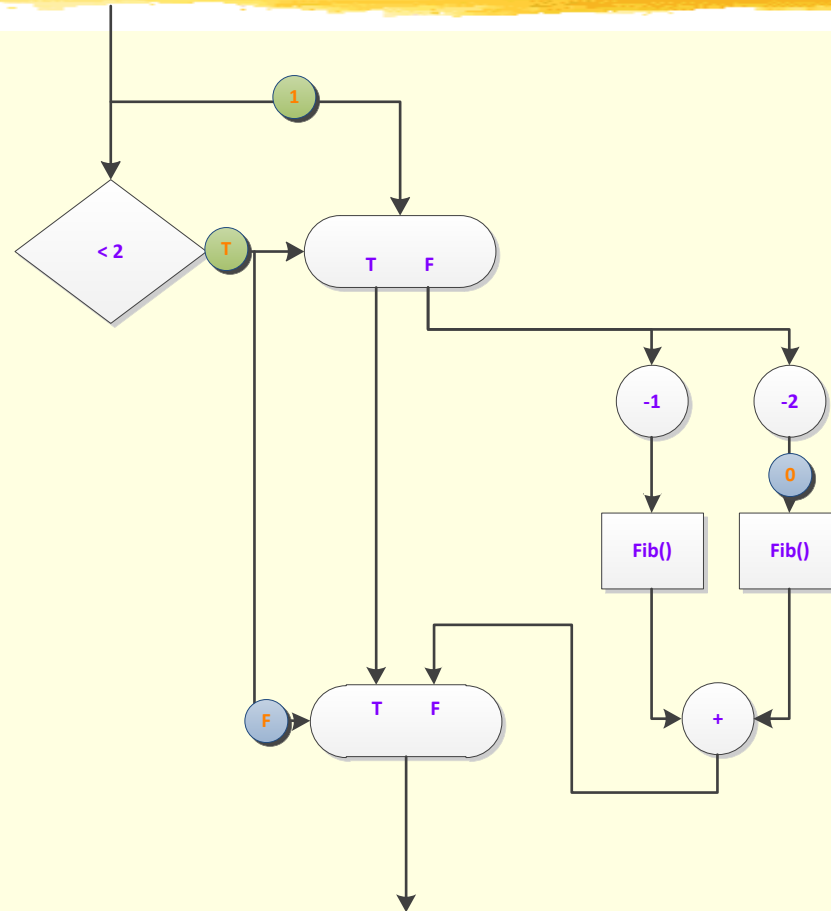
Dynamic Dataflow: Colored Tokens

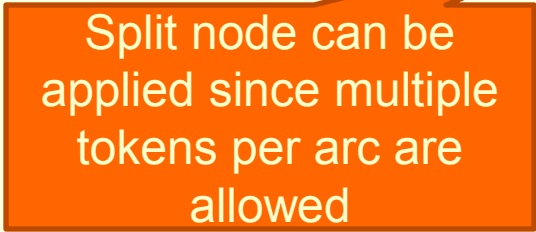
(Thanks to J.Landwehr)



Dynamic Dataflow: Colored Tokens

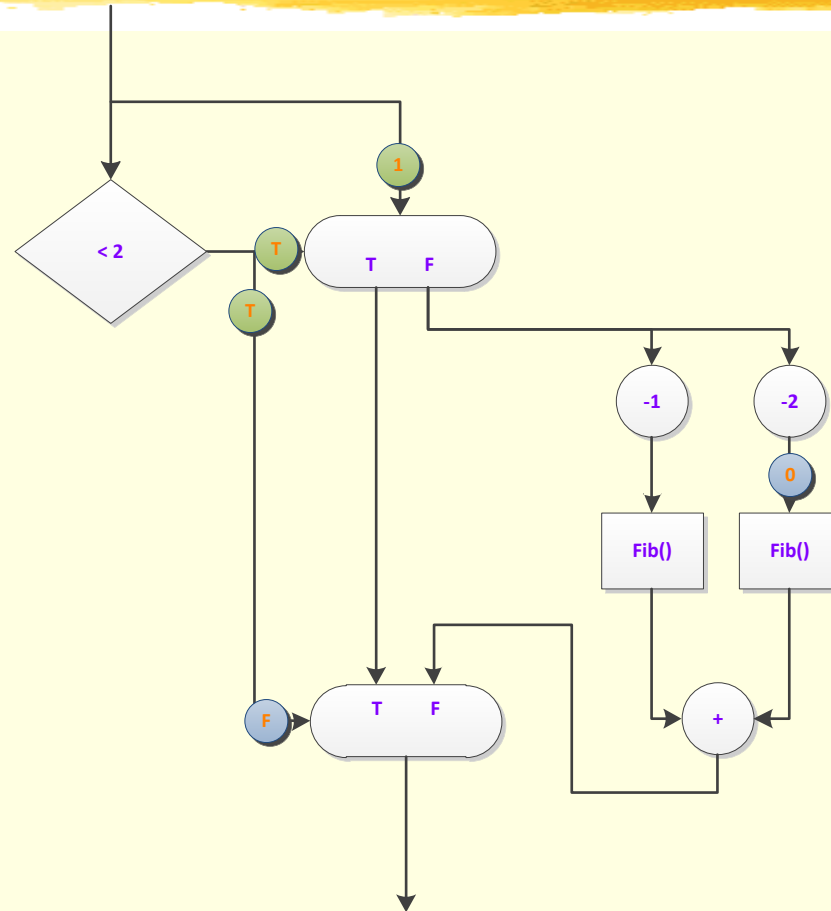
(Thanks to J.Landwehr)





Dynamic Dataflow: Colored Tokens

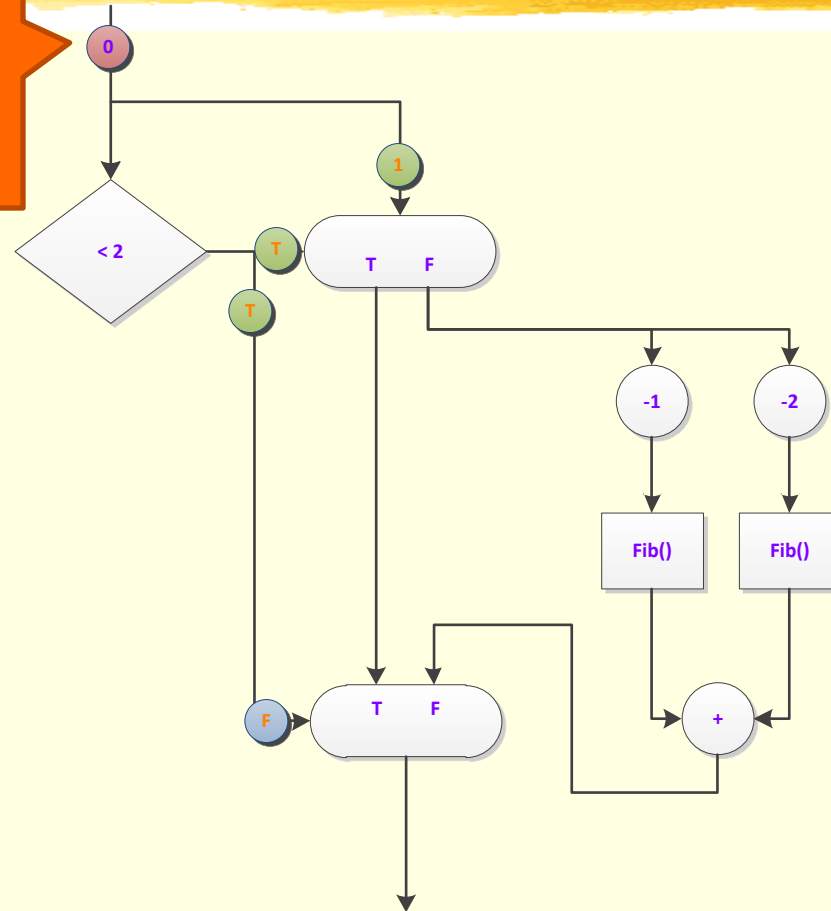
(Thanks to J.Landwehr)



Dynamic Dataflow: Colored Tokens

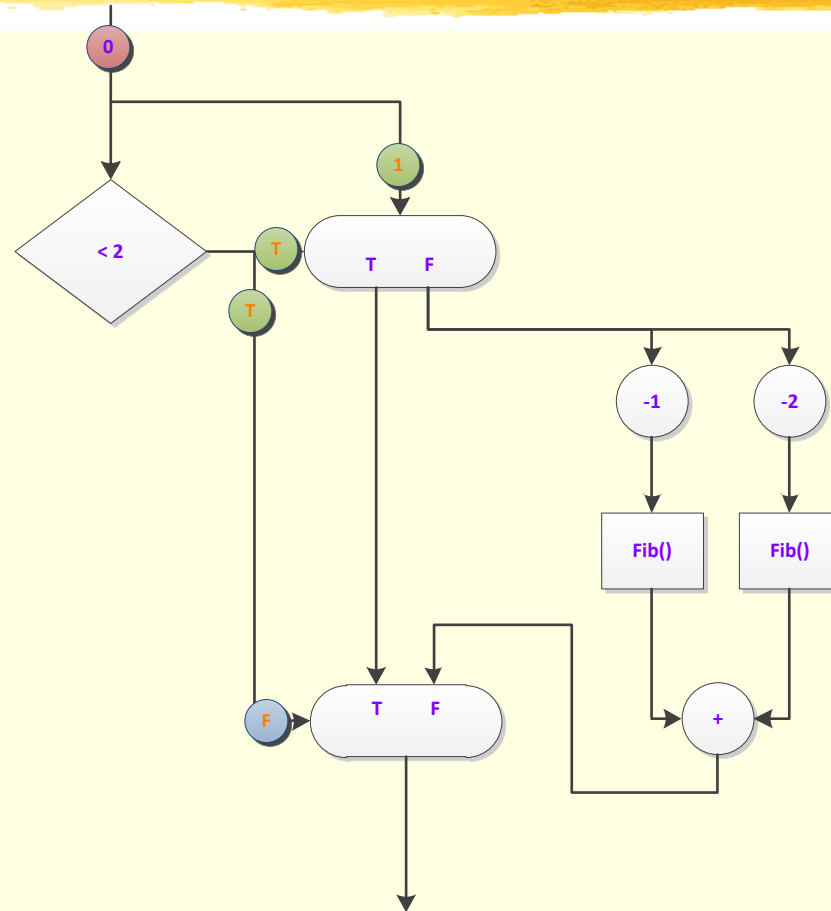
(Thanks to J.Landwehr)

Another new color
chosen on recursion



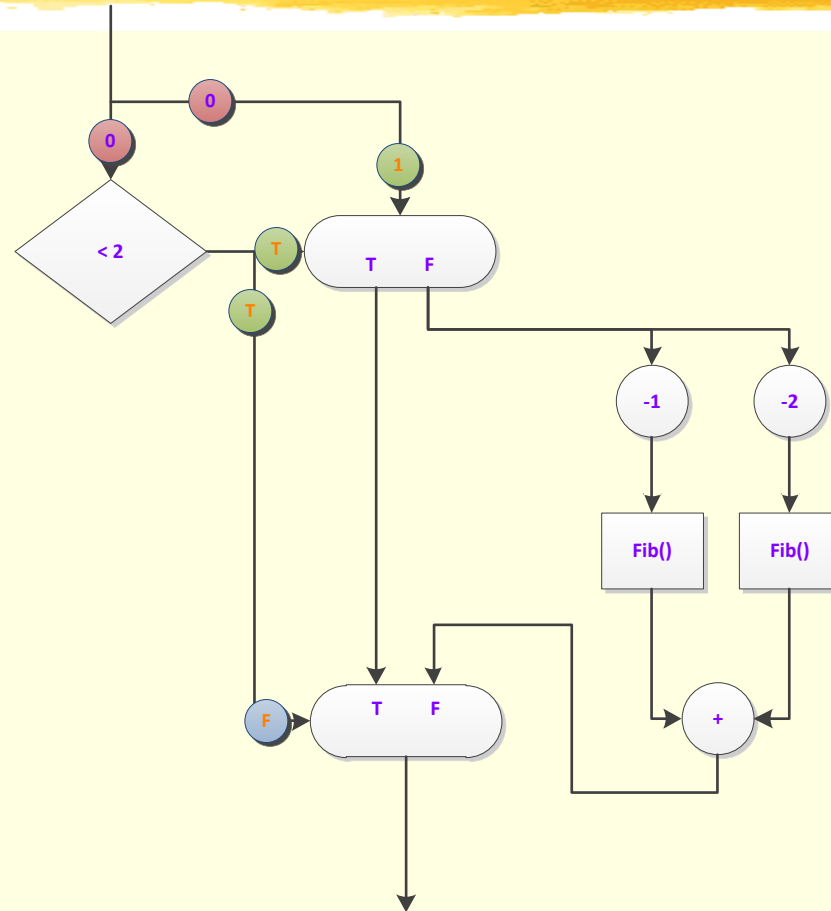
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



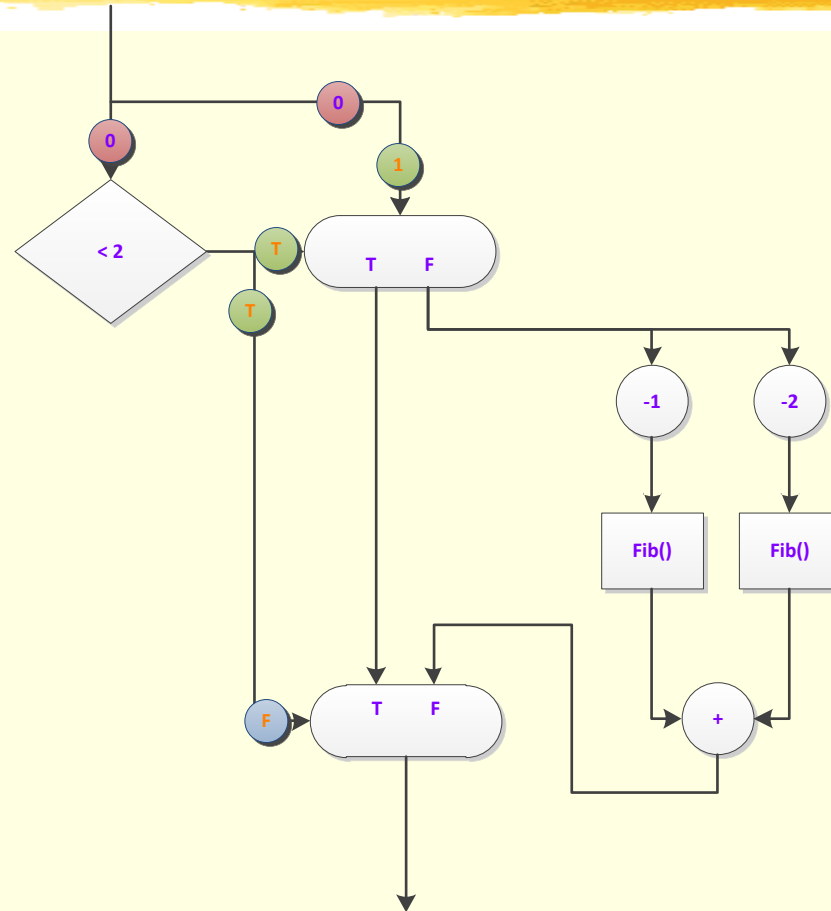
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



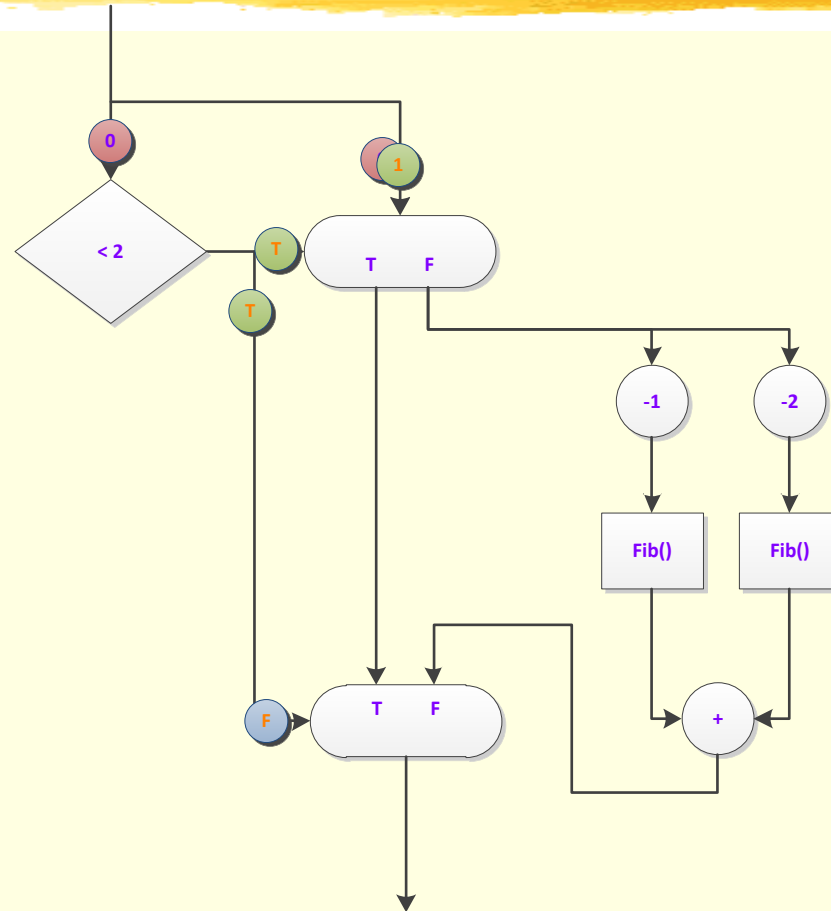
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



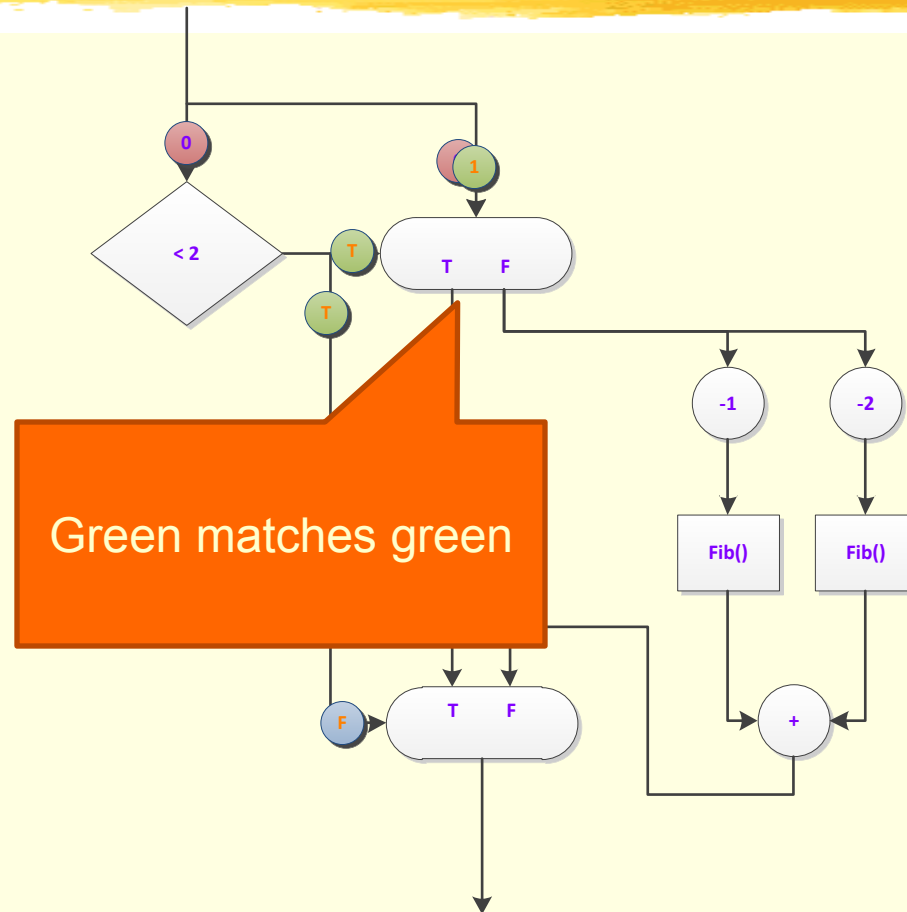
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



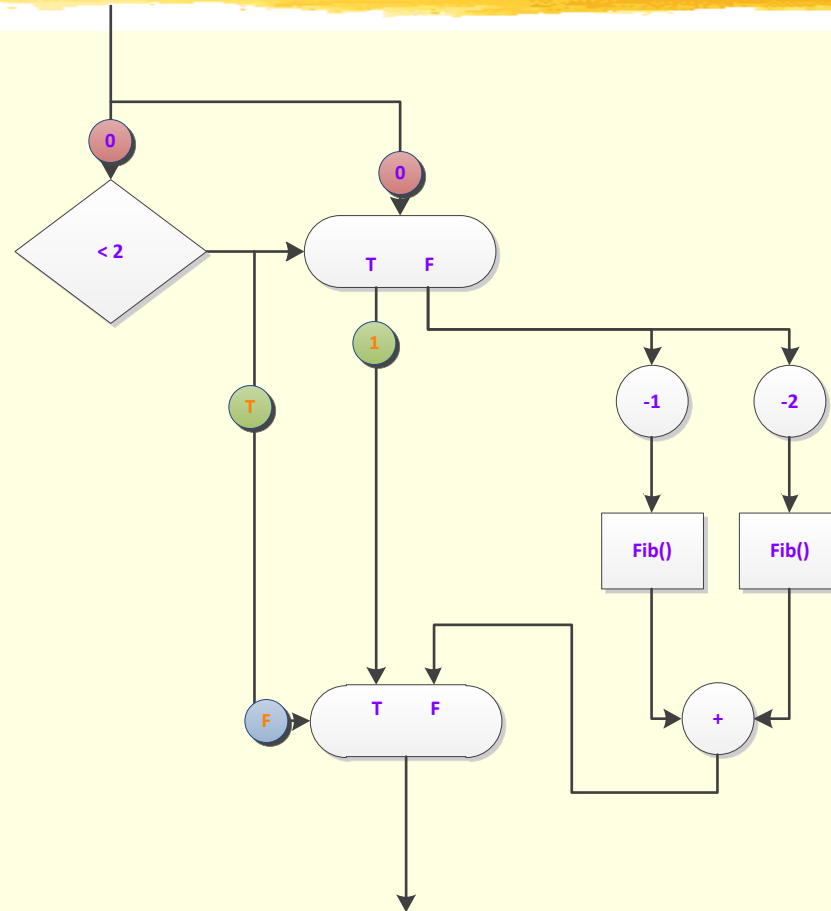
Dynamic Dataflow: Colored Tokens

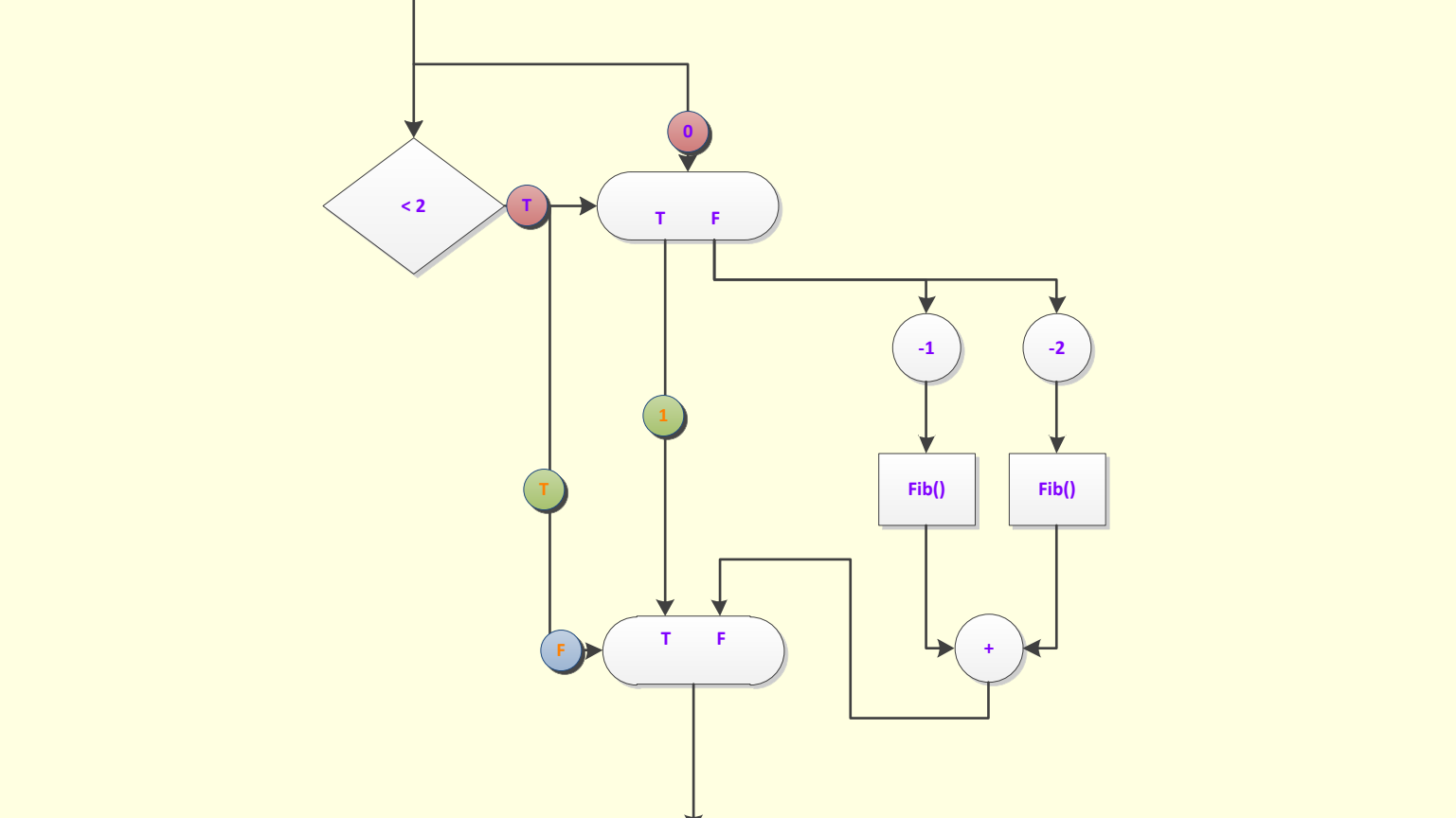
(Thanks to J.Landwehr)



Dynamic Dataflow: Colored Tokens

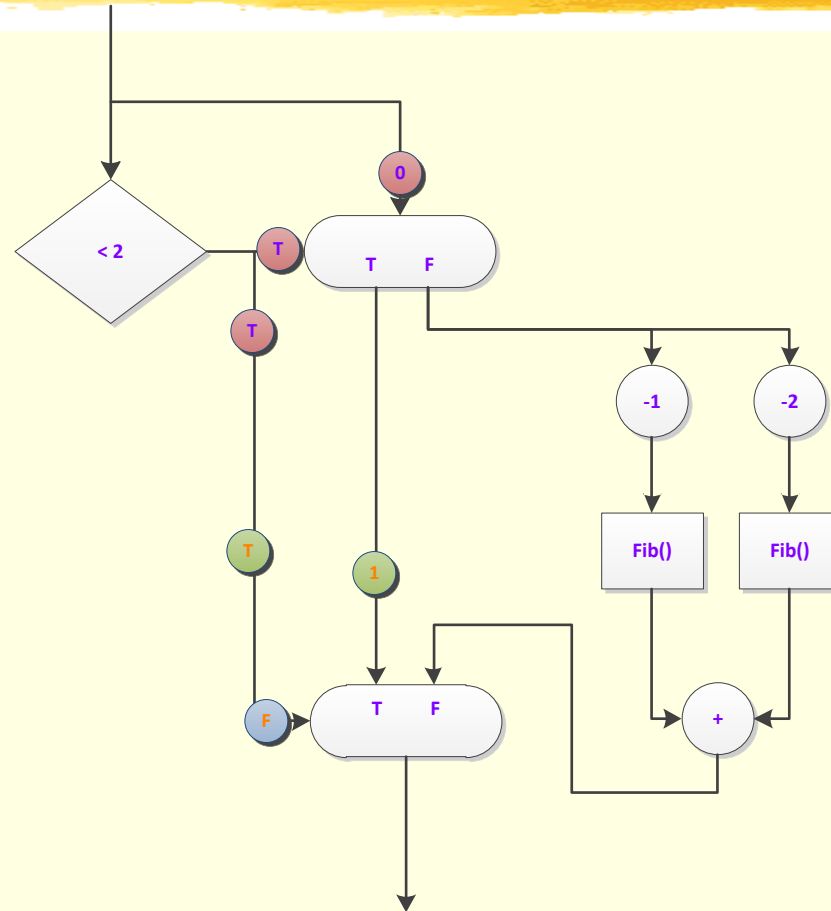
(Thanks to J.Landwehr)

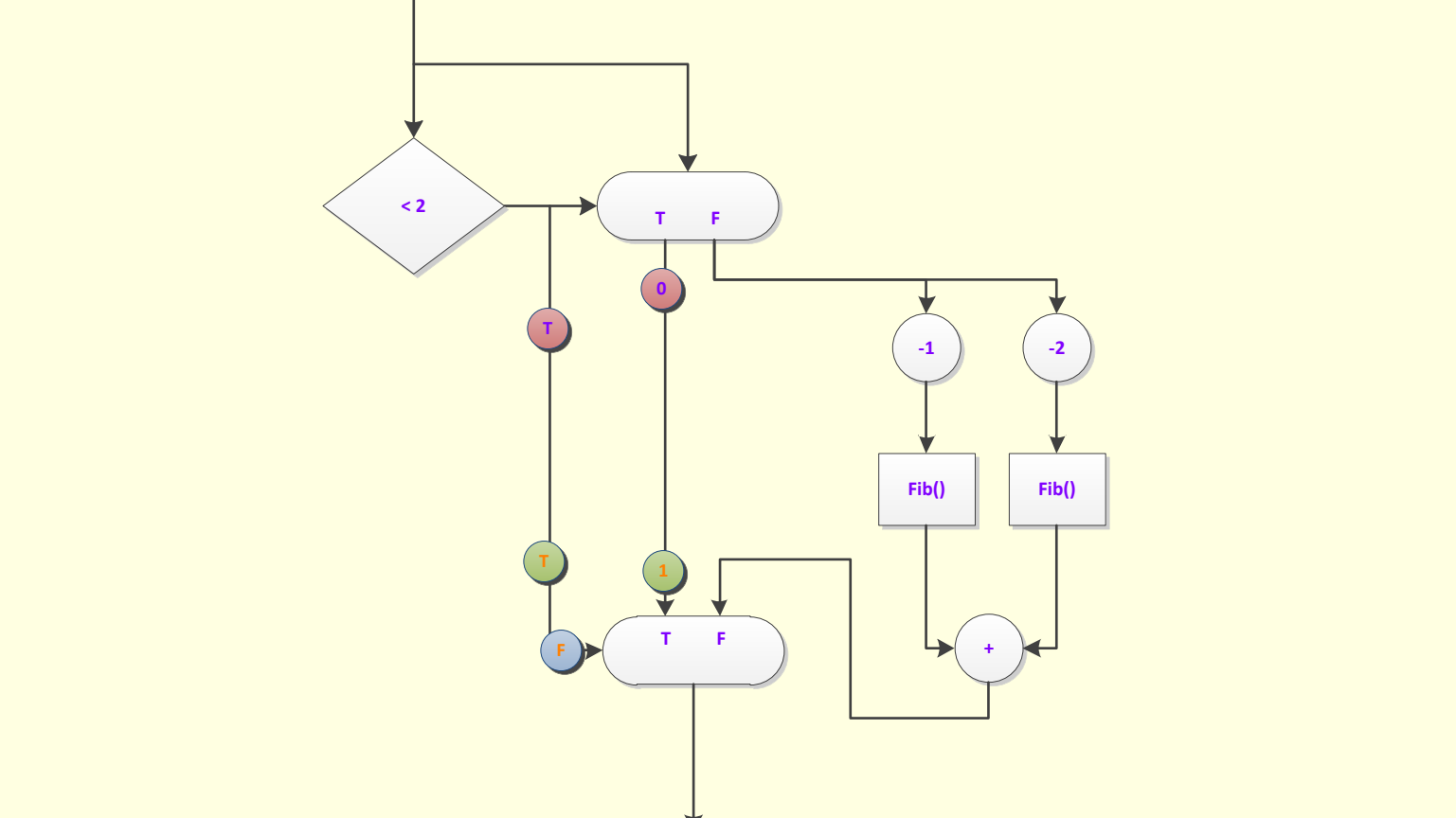




Dynamic Dataflow: Colored Tokens

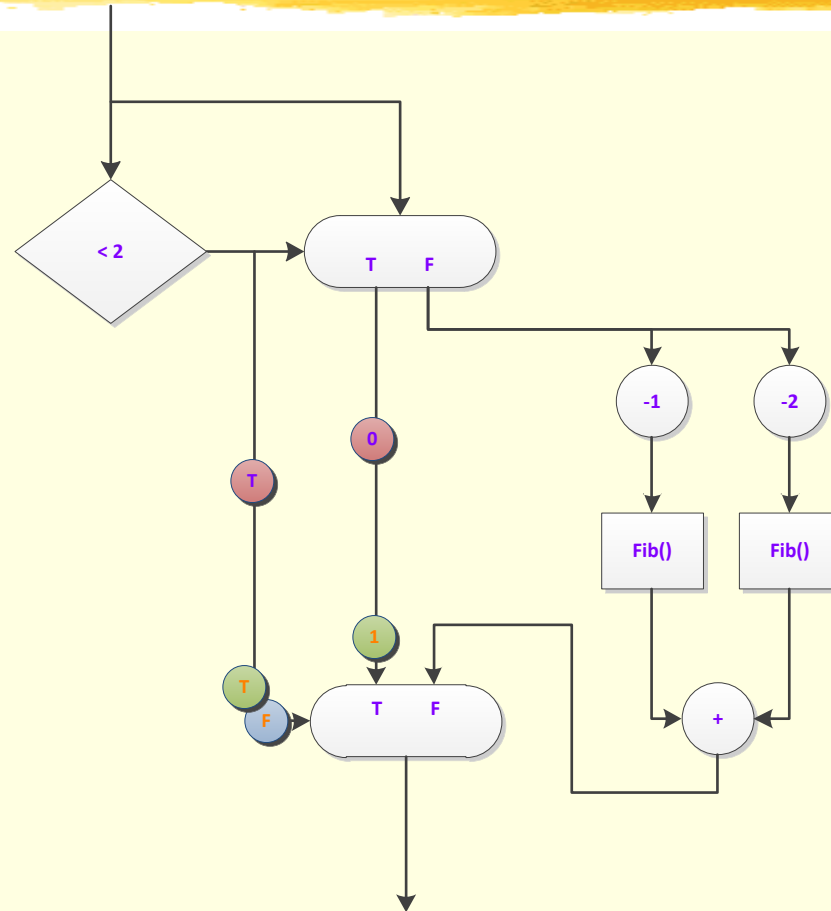
(Thanks to J.Landwehr)





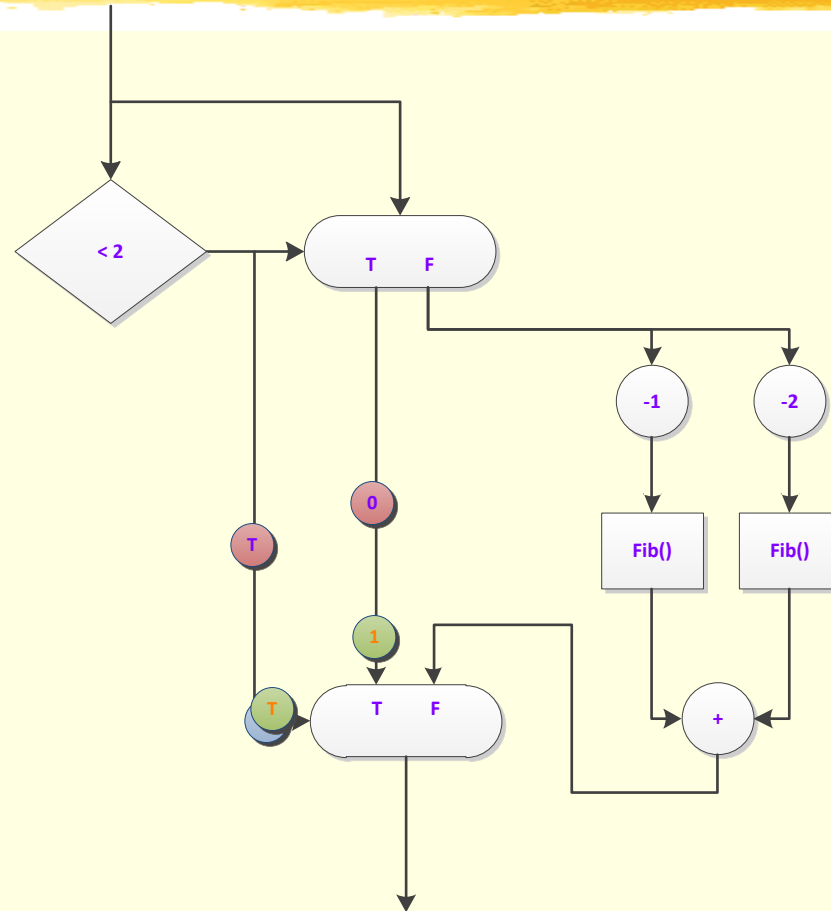
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



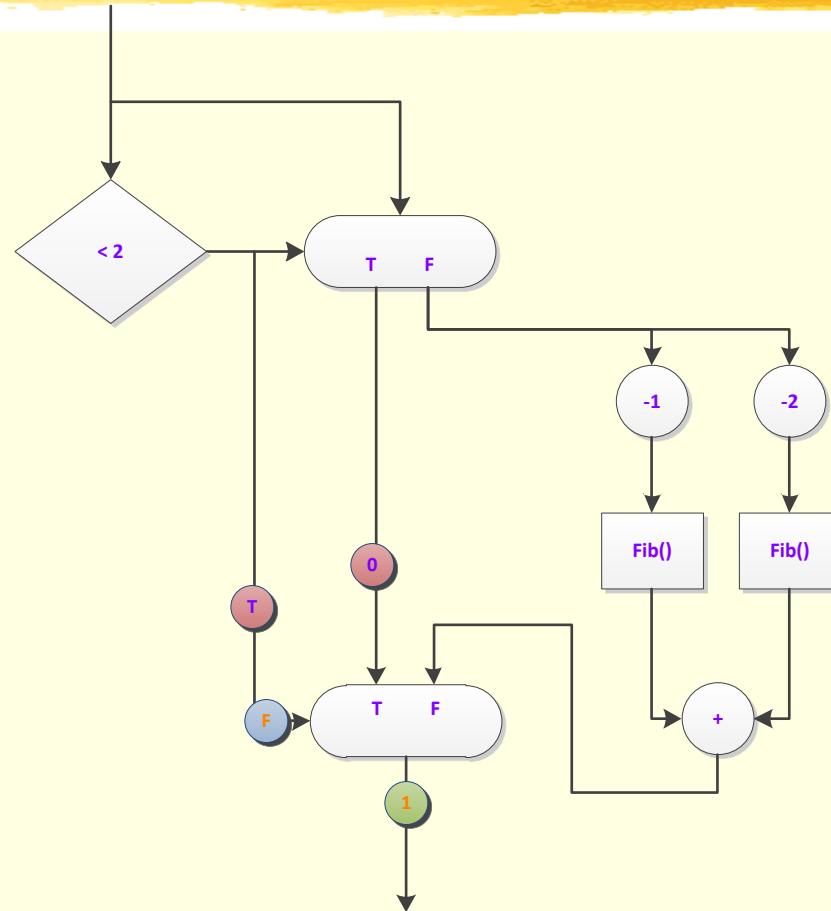
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



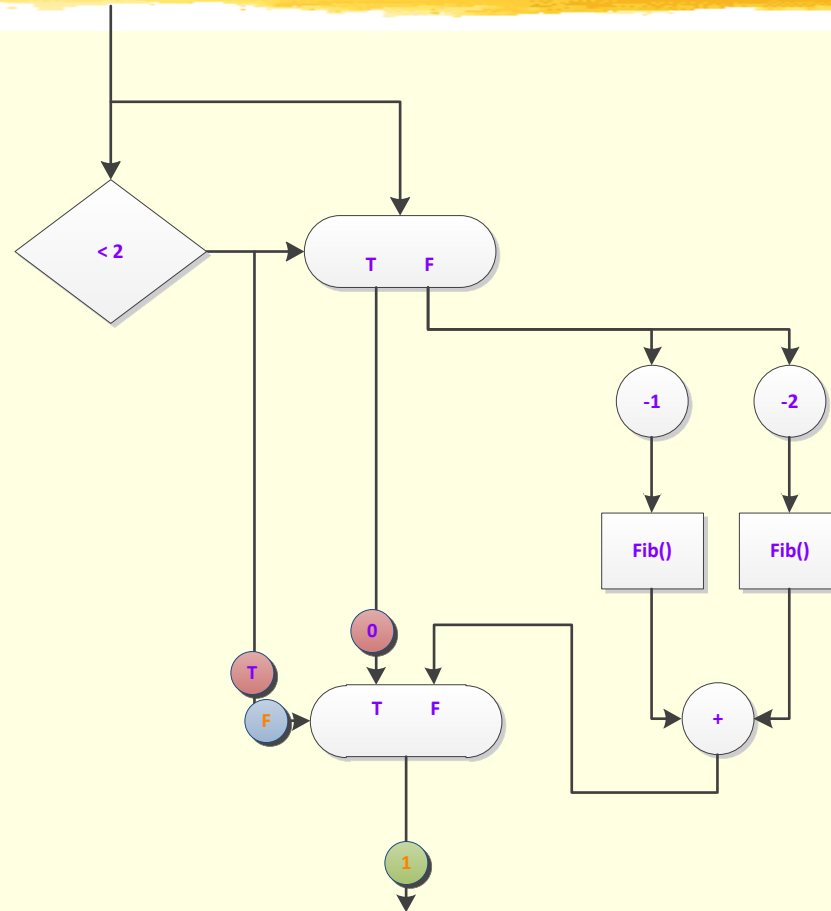
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



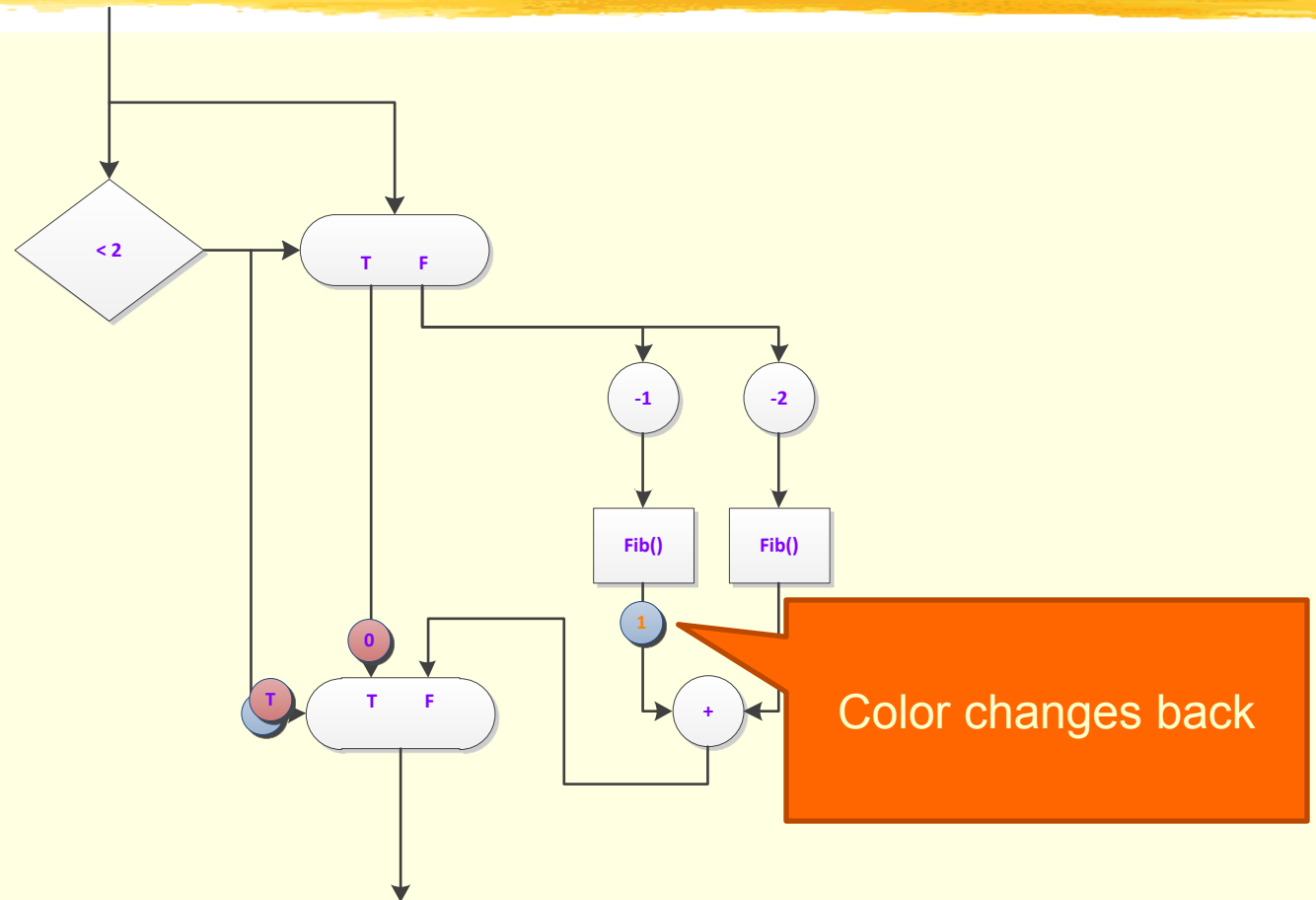
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



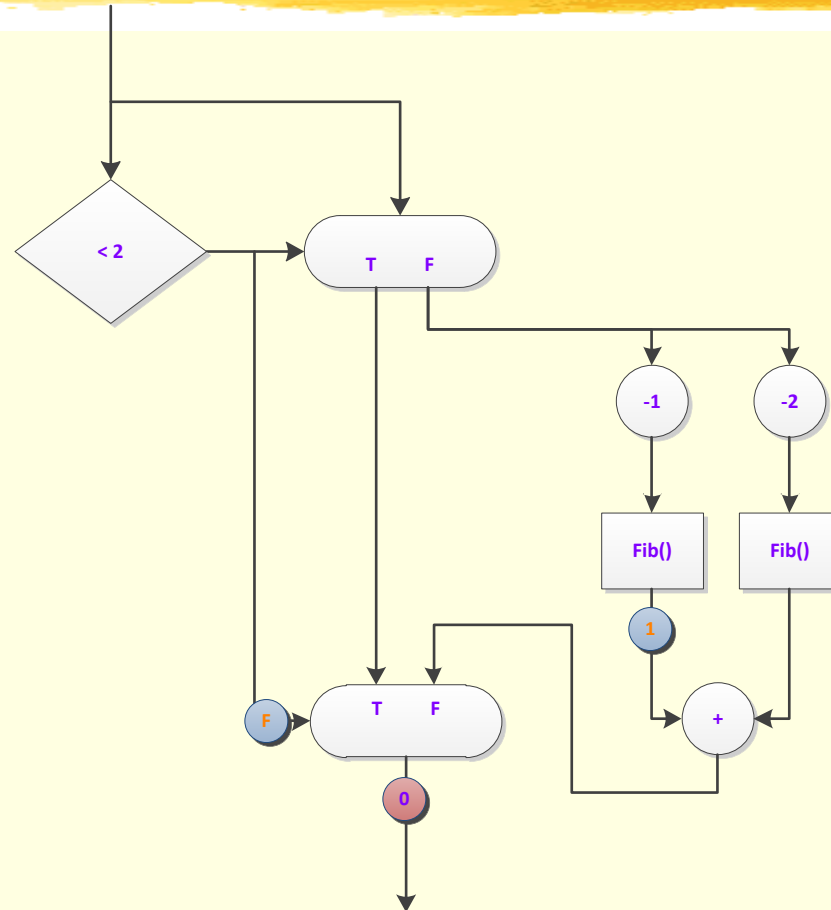
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



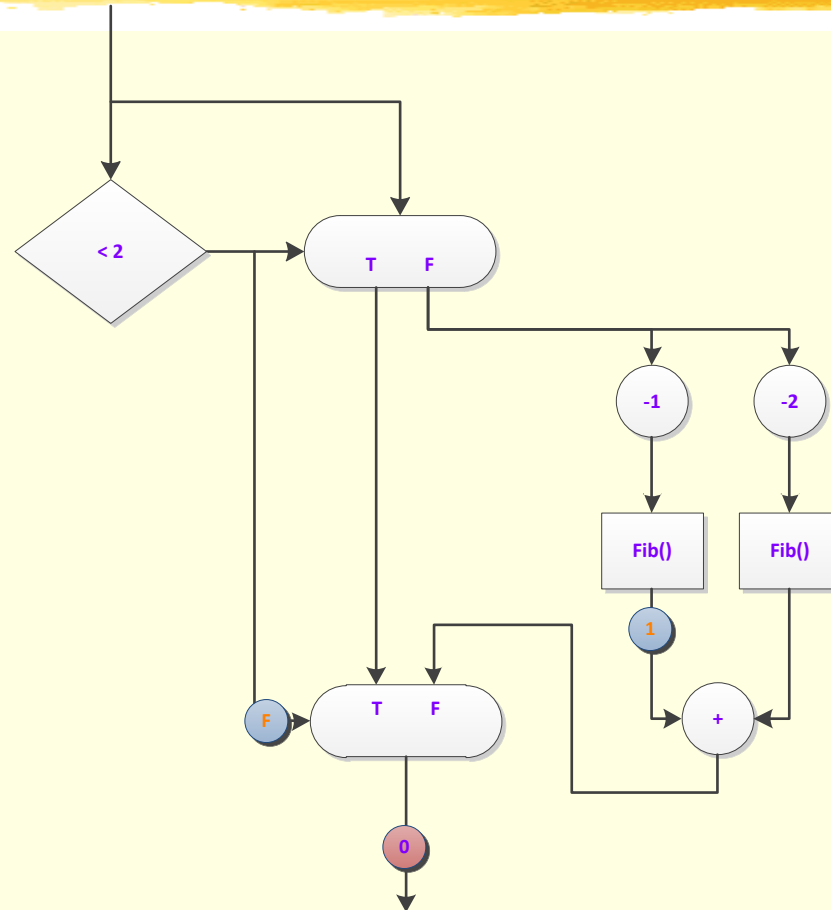
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



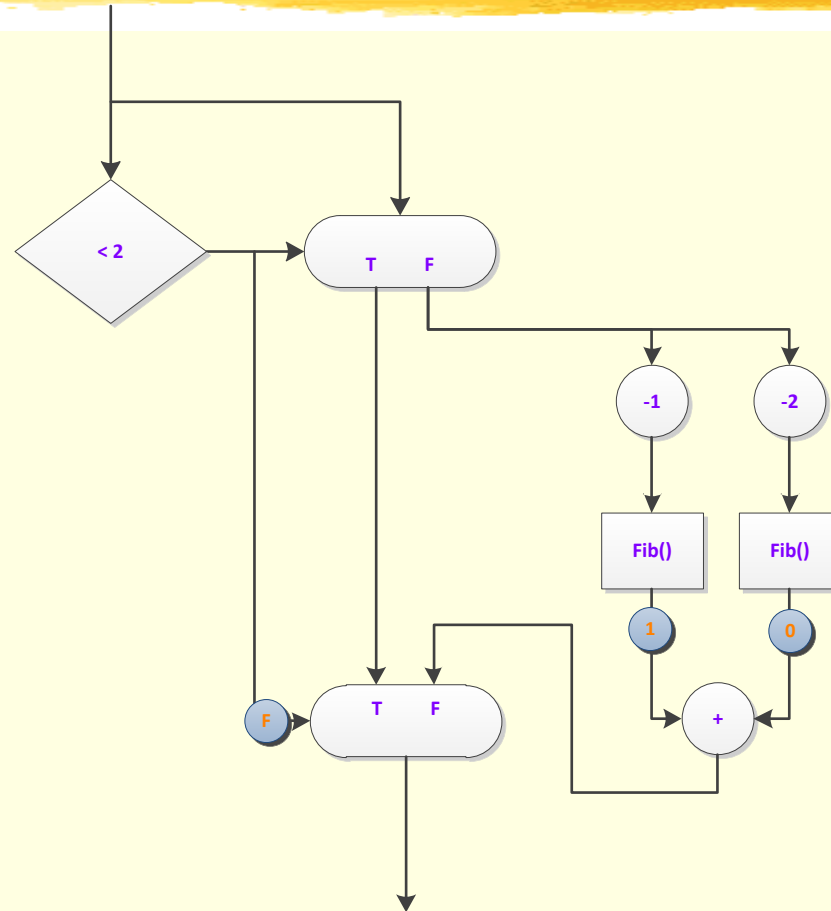
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



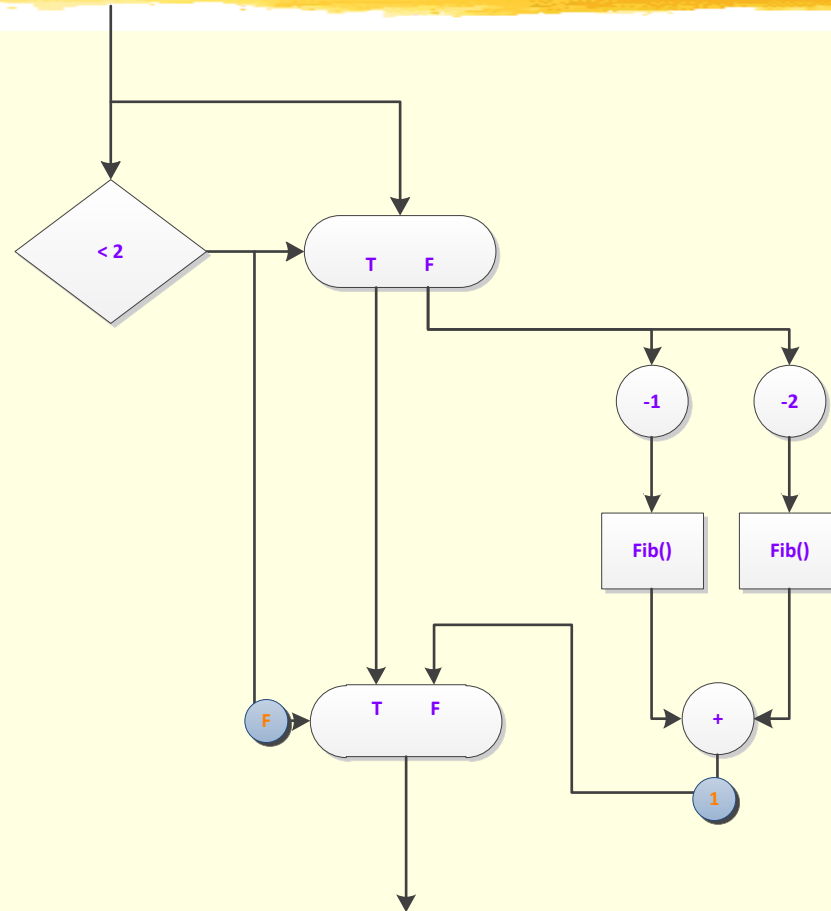
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



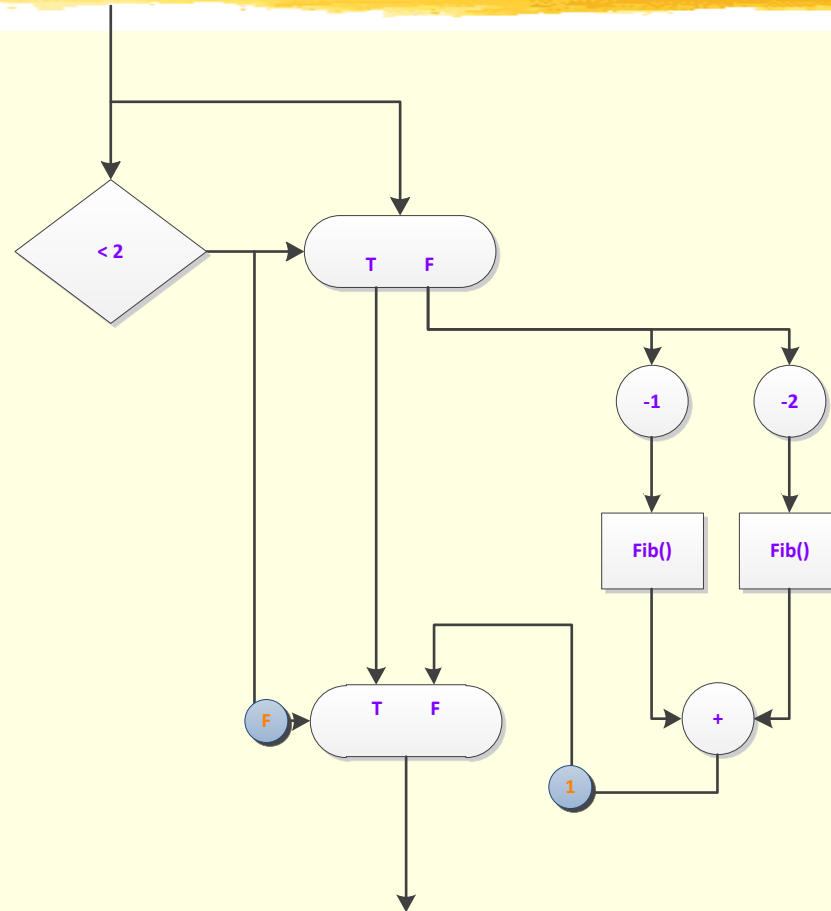
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



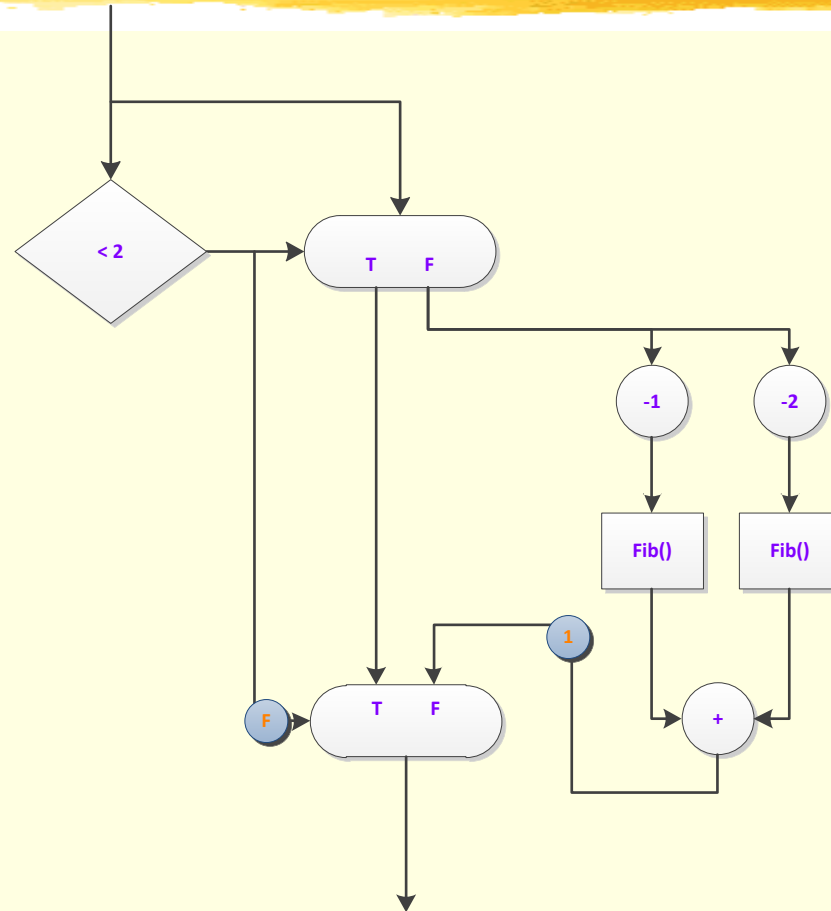
Dynamic Dataflow: Colored Tokens

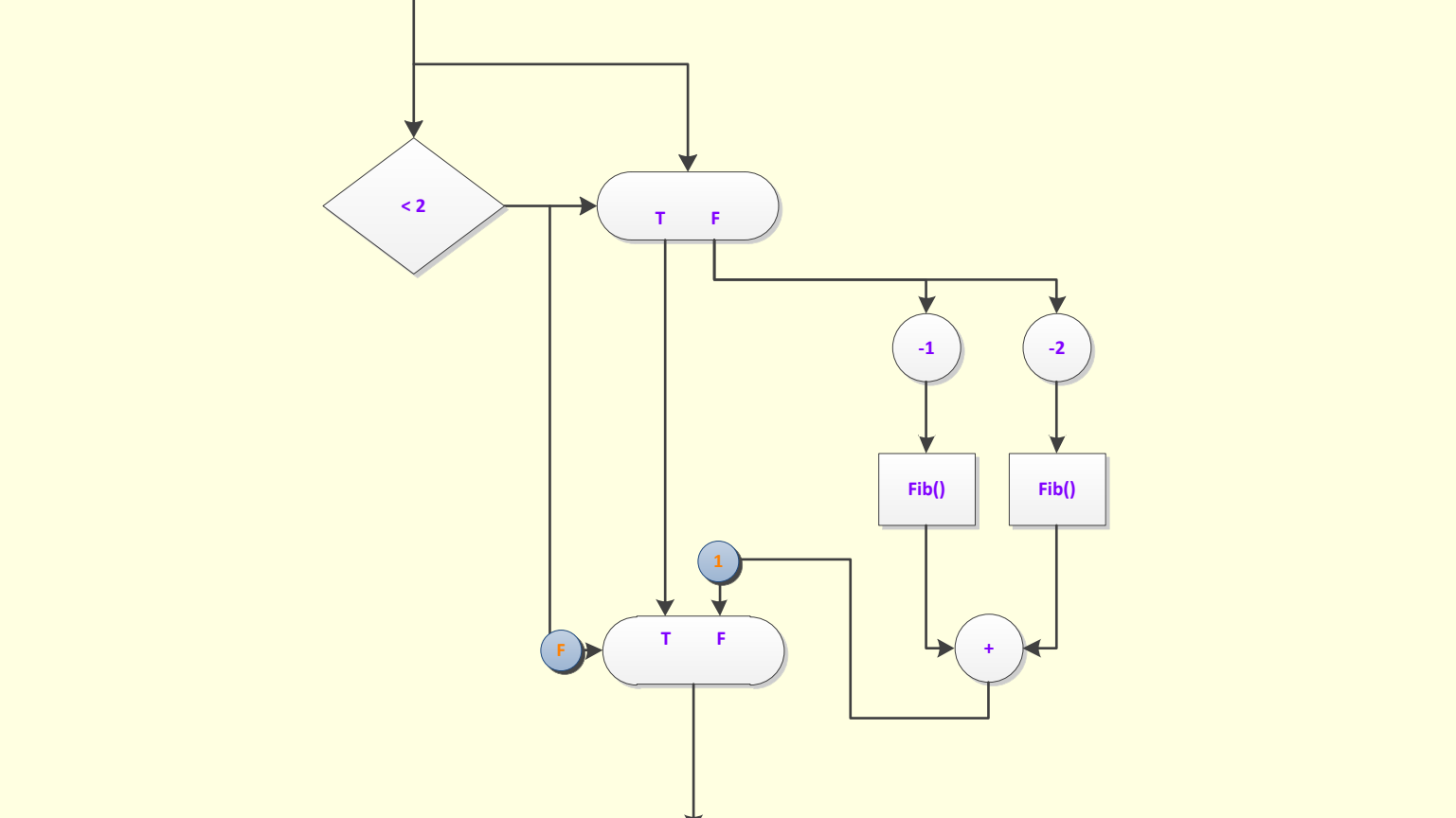
(Thanks to J.Landwehr)



Dynamic Dataflow: Colored Tokens

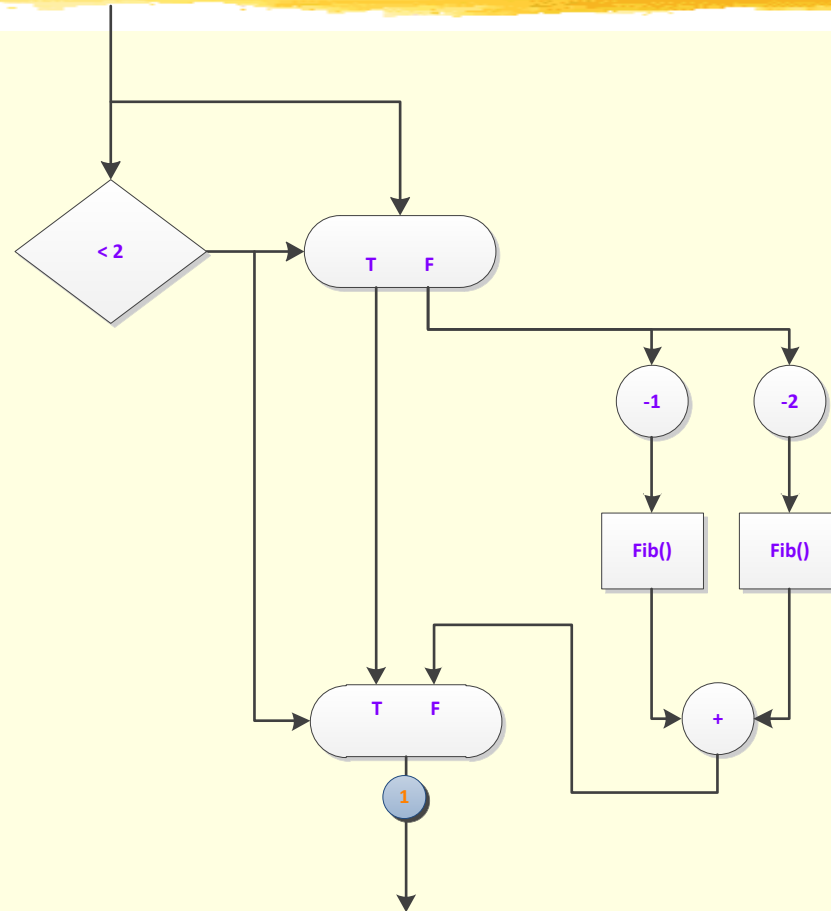
(Thanks to J.Landwehr)





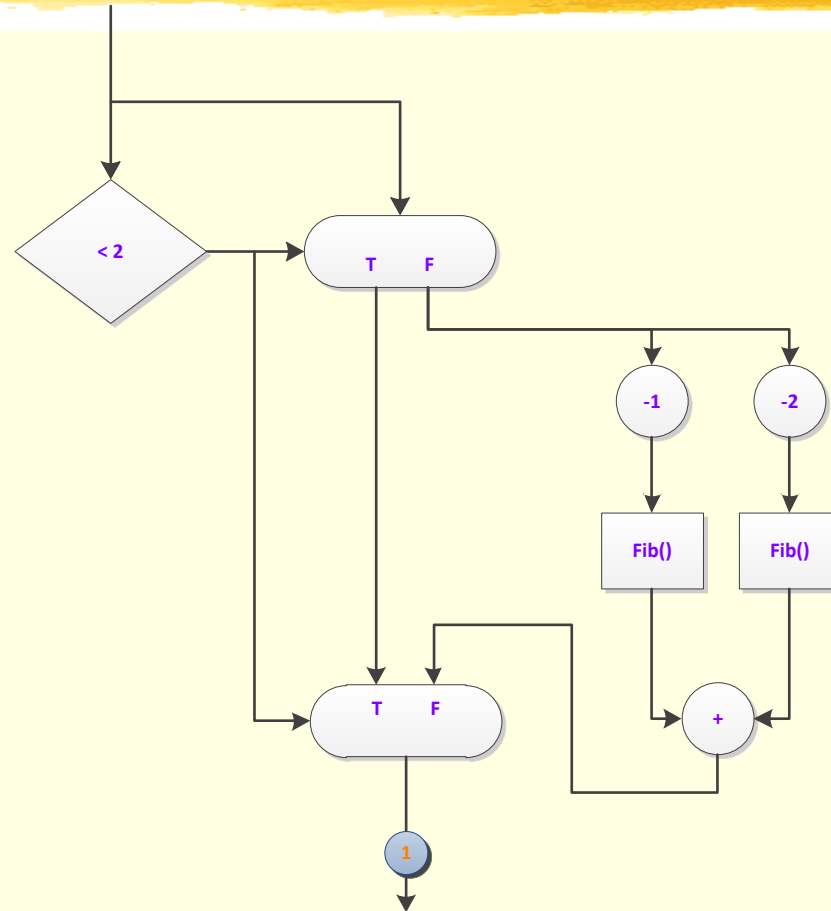
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



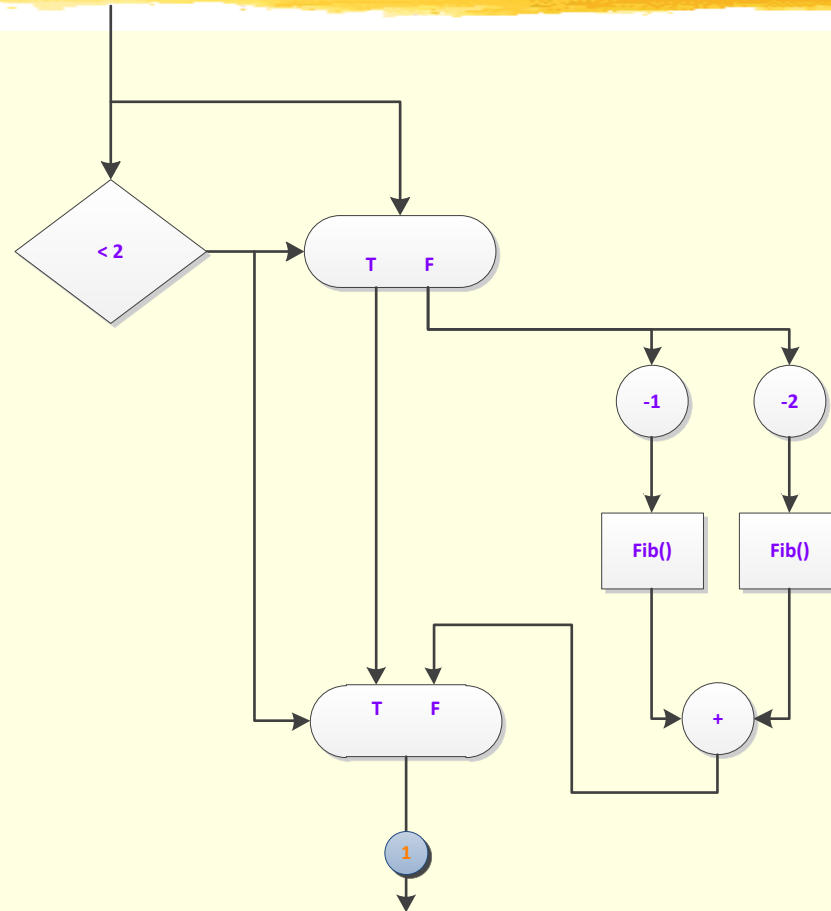
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



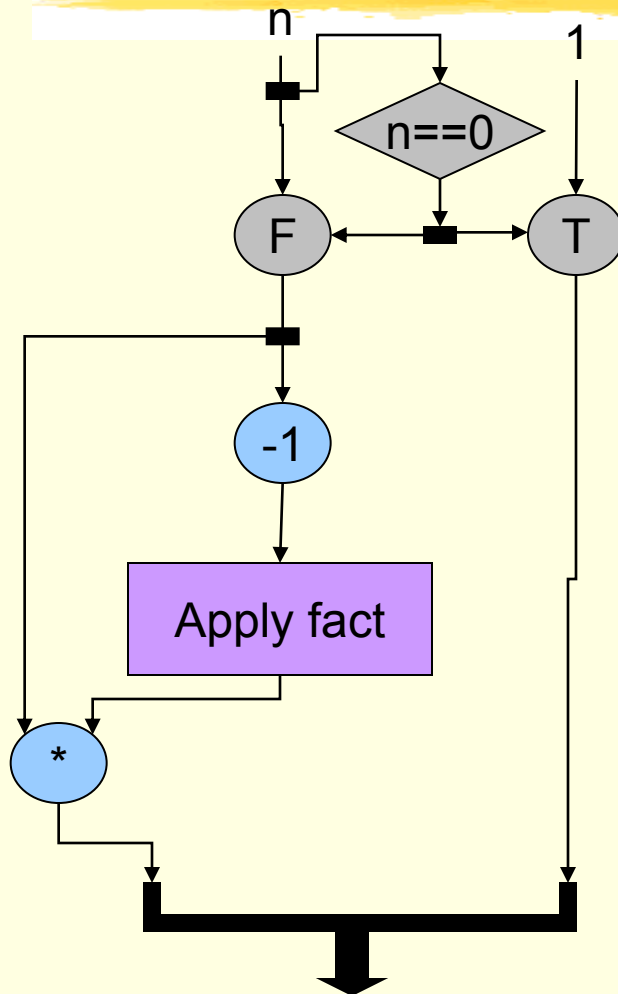
Dynamic Dataflow: Colored Tokens

(Thanks to J.Landwehr)



Analysis: dynamic dataflow can handle arbitrary recursion, but what are its weaknesses? Token matching and color choosing on recursion...

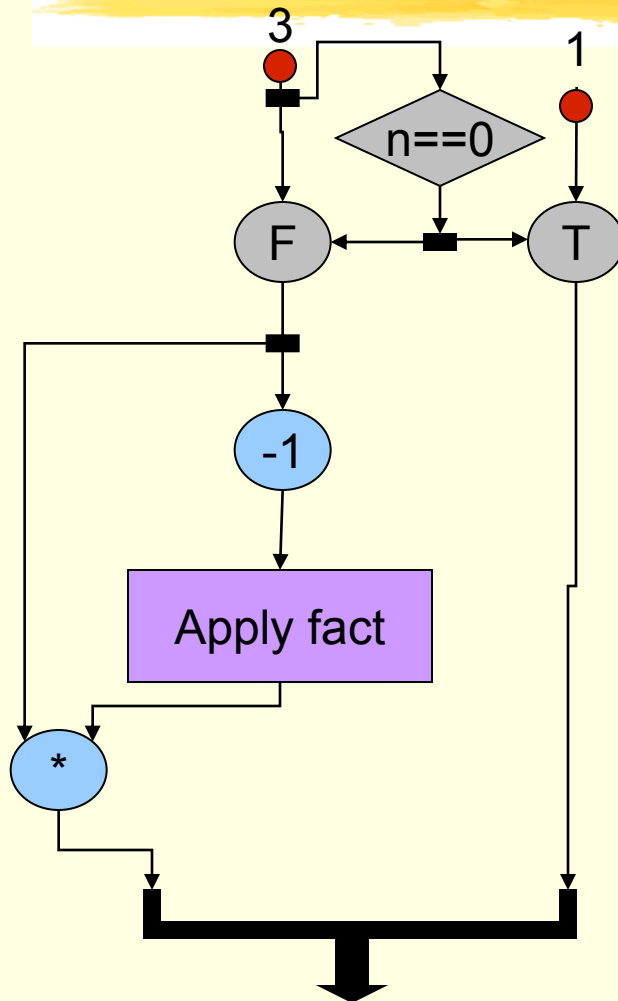
The Normal Version – Dynamic Dataflow



fact(3)

```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

The Normal Version – Dynamic Dataflow



fact(3)

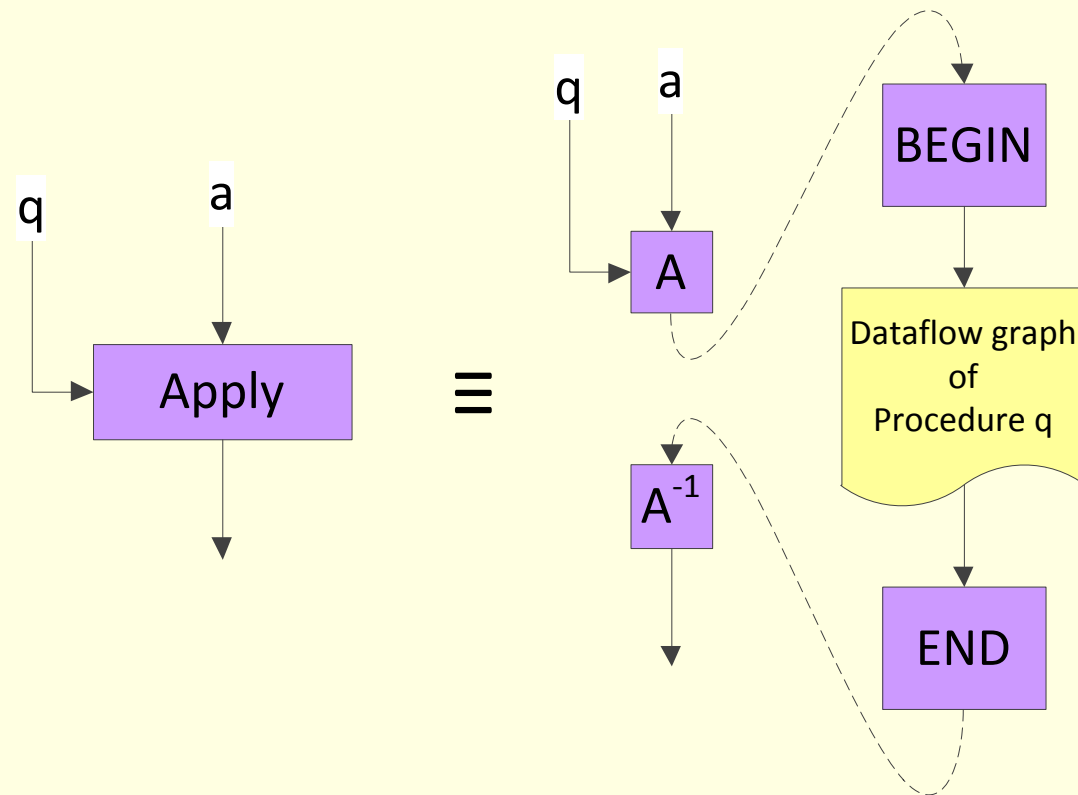
```
long fact(n){  
    if(n == 0) return 1;  
    else return n * fact(n-1);  
}
```

Example of Dynamic Dataflow

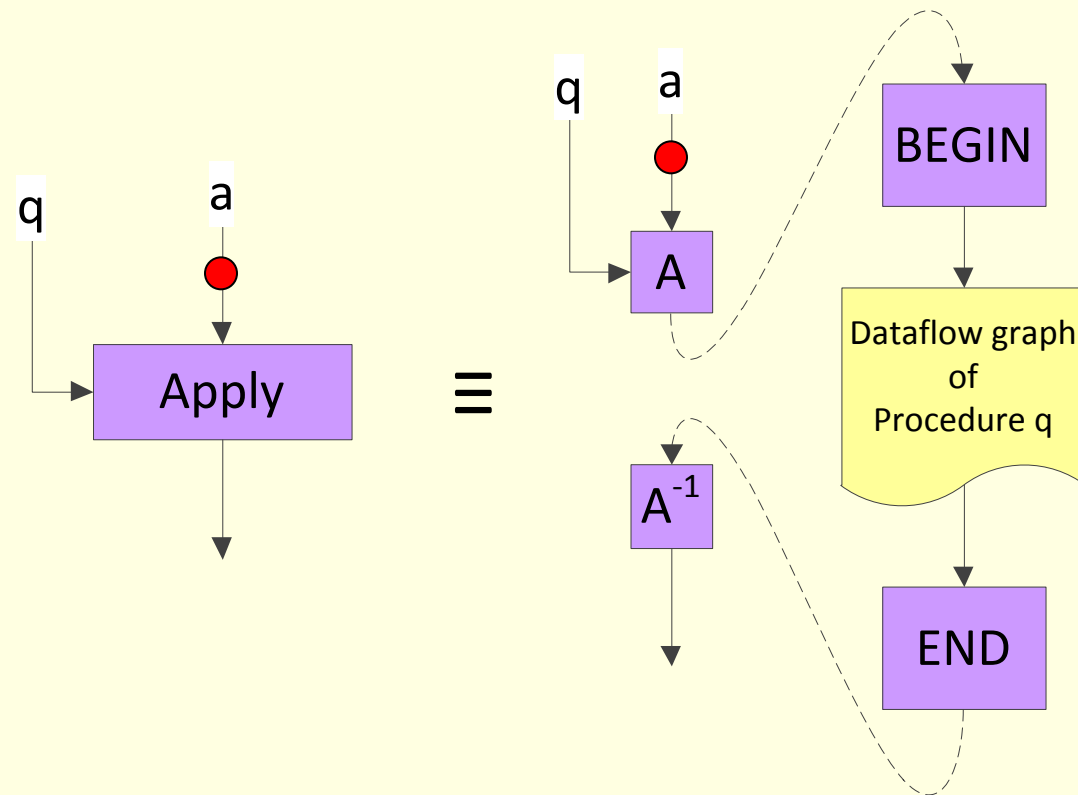
A horizontal yellow brushstroke with a textured, painterly appearance, spanning the width of the slide below the title.

- Example A
- Example B

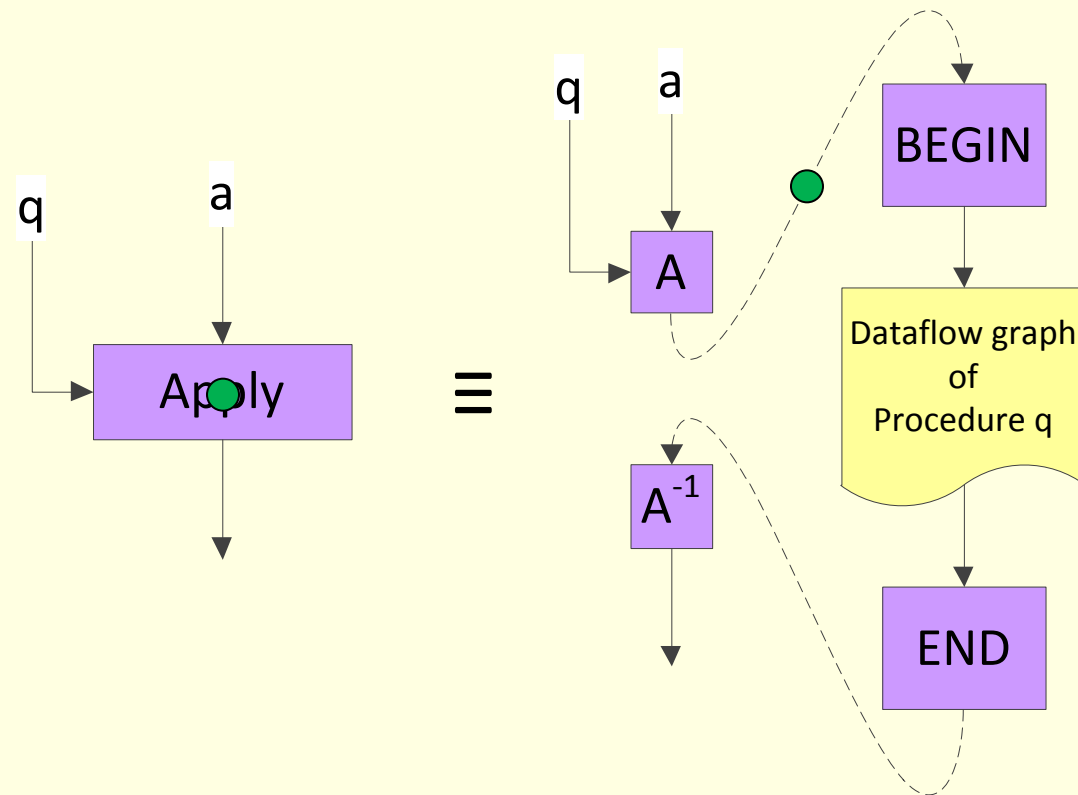
The Apply Operator



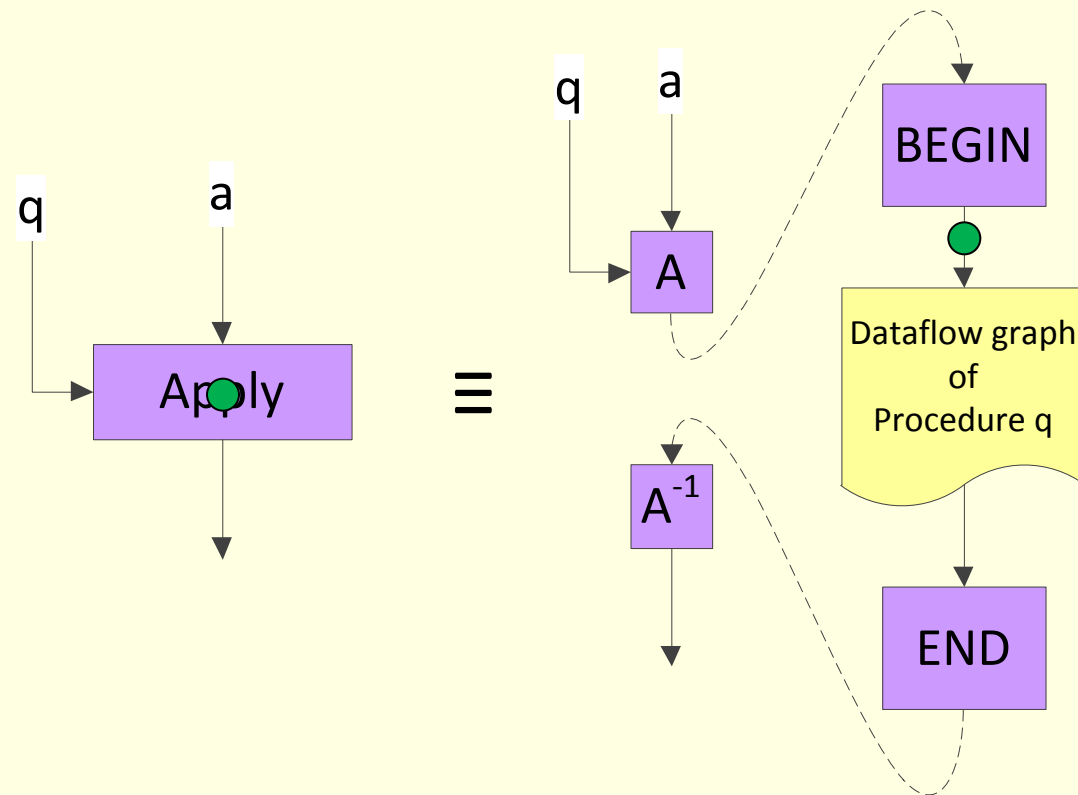
The Apply Operator



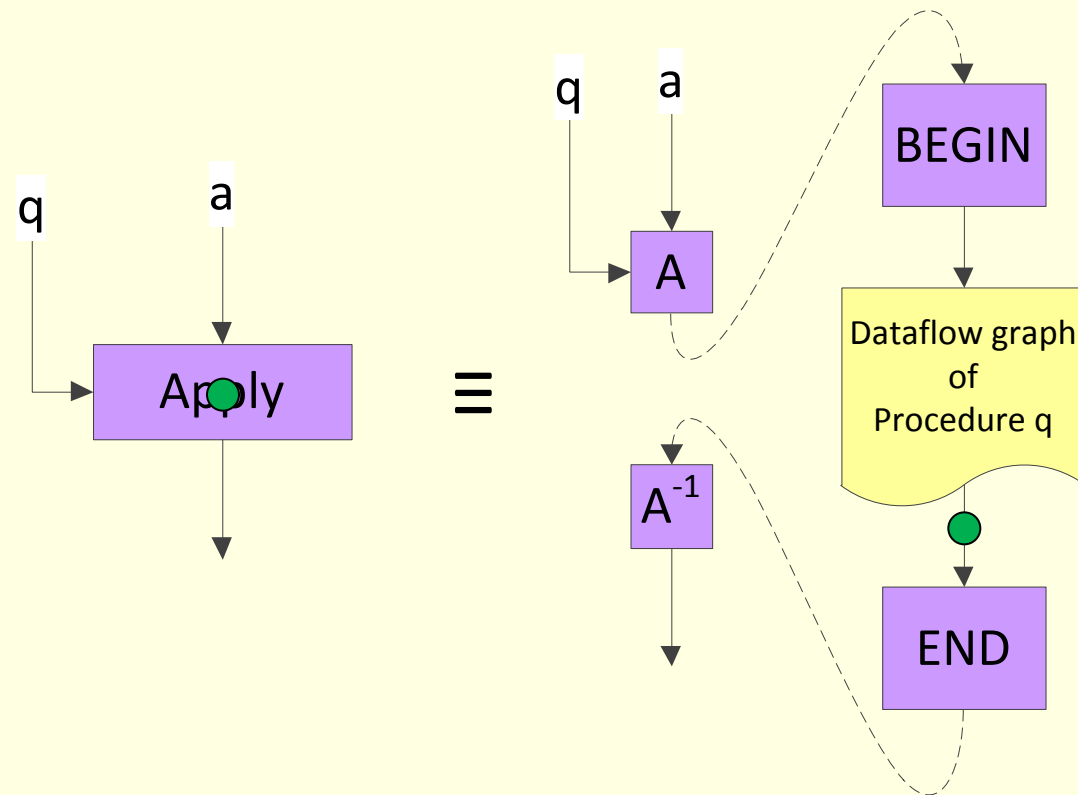
The Apply Operator



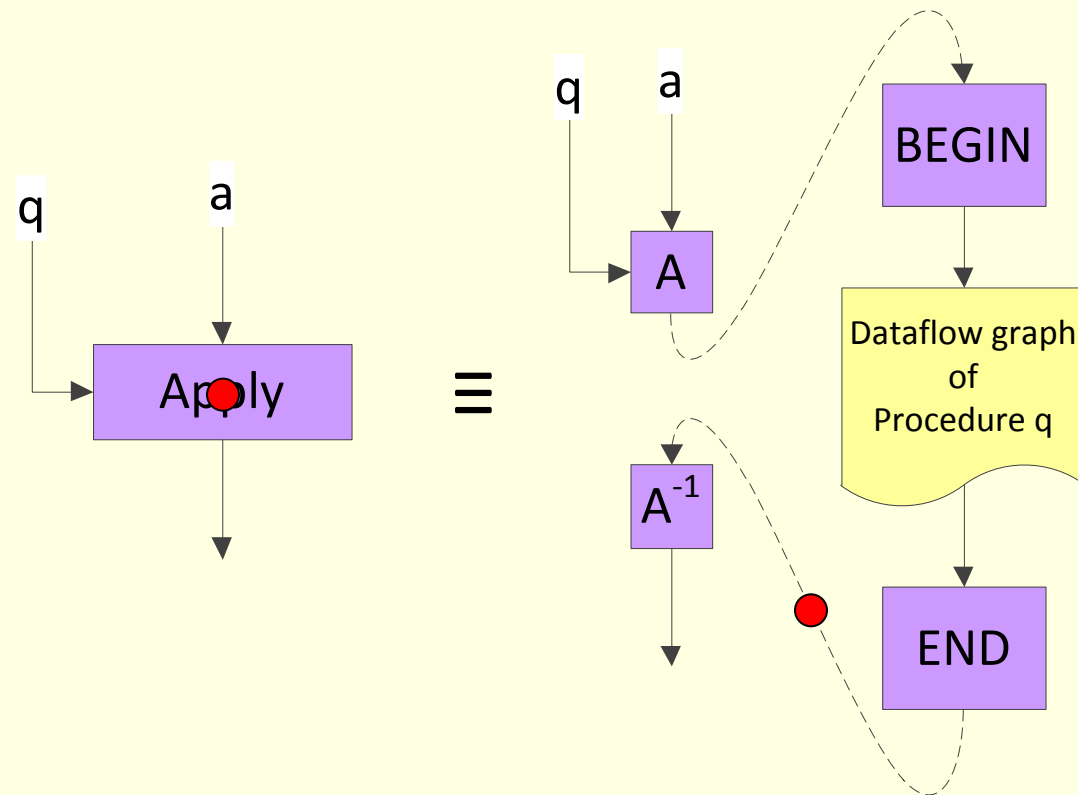
The Apply Operator



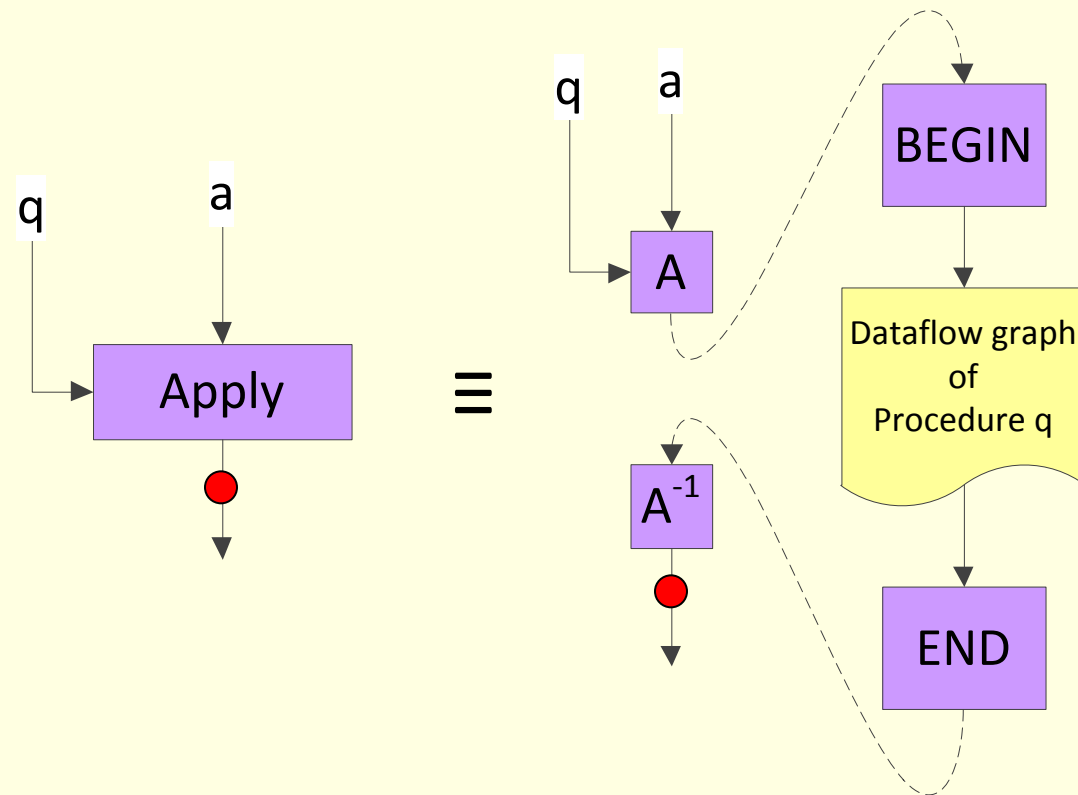
The Apply Operator



The Apply Operator



The Apply Operator



Dynamic Dataflow



- Advantages

- More Parallelism
 - Handle arbitrary recursion

- Disadvantages

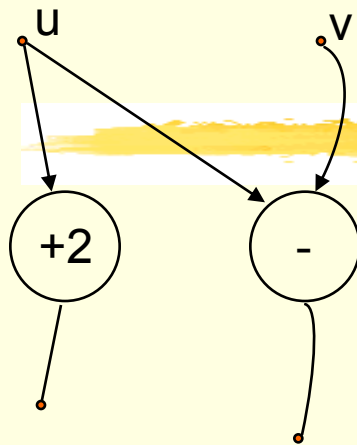
- Implementation of the token tag matching unit

- Associative Memory would be ideal

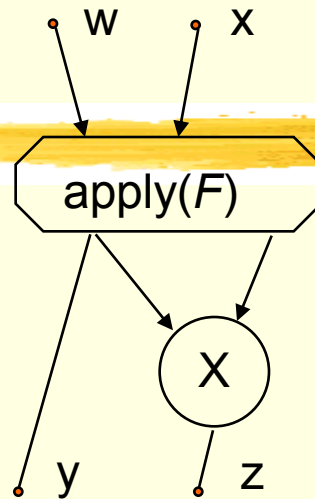
- Not cost effective
 - Hashing is used

Features of Data Flow Computation Model

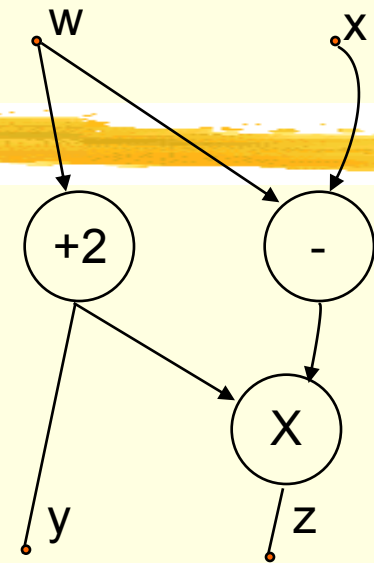
- Not history-sensitive → no real concept of time!
- Semantics (determinate)
- Parallelism



(a)
Function F

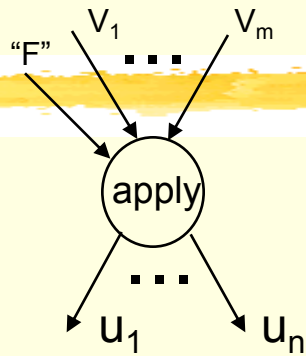


(b)
Function G

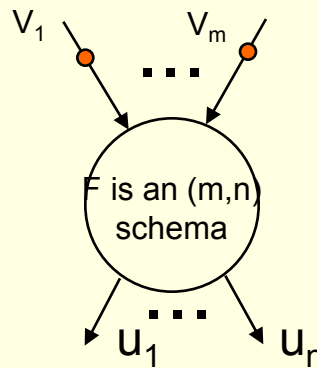
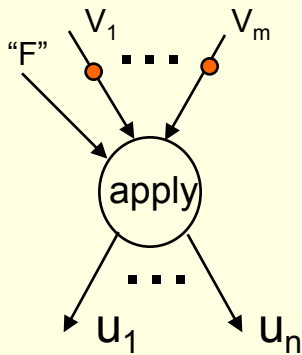


(c)
Function G'

Interpreting function invocation as module substitution



(a) Notation for apply



(b) Firing Rule

The Apply Actor

Concept of Strictness

Intuitive:

a mechanism that requires all arguments to be evaluated before evaluation of the body of a function may begin

Formal :

A function f is strict if

$$f \perp = \perp$$

nonstrict = not strict

Comment on Strict “Apply” Actor



- Advantage - Parallelism
- Call-by-value semantics
- Disadvantage:
 - Lose “substitution” property or “referential transparency” property
 - to avoid it, need strictness analysis

Referential Transparency

“...The only thing that matters about an expression is its value, and any subexpression can be replaced by any other equal in value...”

“...Moreover, the value of an expression is, within certain limits, the same when ever it occurs...”

“Referential Transparency” or “Property of Substitution”

Let f, g be two procedures/functions such that f appears as an application inside of g ; let g' be the procedure obtained from g by substituting the text of f in place of the application;

In the languages which are “referential transparent”, the specification of the function for g will not depend on any terminating property of f , or $g=g'$.

Concept of Non-Strictness



- Lenient evaluation model
- Lazy evaluation model

Memory Model and Dataflow



- What is “memory” in dataflow model ?
- What is “memory model” under dataflow model ?
- Concept of functional programming
- Concept of *single-assignment* and single-assignment programming languages

The I Structures



- Single Assignment Rule and Complex Data structures
 - Consume the entire data structure after each access
- The Concept of the I Structure
 - Only consume the entry on a write
 - A data repository that obeys the Single Assignment Rule
 - Written only once, read many times
- Elements are associated with status bits and a queue of deferred reads

I-Structure Memory



- Extending dataflow abstract Machine model with I-Structure Memory
- Use a data structure before it is completely defined
- Incremental creating or reading of data structures

Dataflow

The I Structures



- An element of a I-structure becomes defined on a write and it only happens only once
- At this moment all deferred reads will be satisfied
- Use a I-structure before it is completely defined
- Incremental creating or reading of data structures
- Lenient evaluation model - revisited

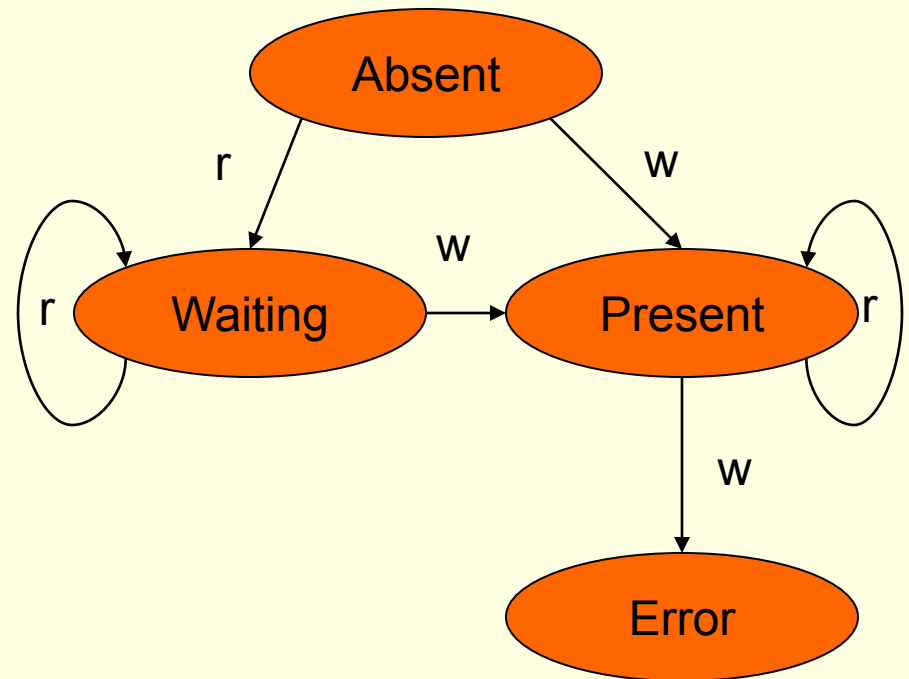
The I Structures: state transitions

- States

Present: The element of an I-structure (e.g. $A[i]$) can be read but not written

Absent: The element has been attempted to be read but the element has not been written yet (initial state)

Waiting: At least one read request has been deferred



I Structures



- Elementary
 - Allocate: reserves space for the new I-Structure
 - I-fetch: Get the value of the new I-structure (deferred)
 - I-store: Writes a value into the specified I structure element
- Used to create construct nodes:
 - SELECT
 - ASSIGN